Testing the Monthly Effect of Agricultural Futures Markets with Stochastic Dominance

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Abstract: This study uses a bootstrap-based test of Linton, Maasoumi and Whang (2005) (LMW test) based on stochastic dominance (SD) theory to examine the monthly effects for four active agricultural futures in the U.S. markets: corn, soybeans, soybean meal and wheat. We find returns in October for corn, April for soybeans and August for soybean meal and wheat futures dominate returns of other months. In addition, allocating part of investors' portfolio in riskless assets enables us to distinguish better the performance among months for various futures.

1. Introduction

Market efficiency becomes a prominent topic for empirical research due to the introduction of the analysis of the capital market efficiency and the assertion of the Efficient Market Hypotheses (EMH) by Fama (1970). Many empirical studies examine the behavior of financial asset prices and document the existence of calendar anomalies in stock markets.[4] One of the most well-known calendar anomalies shows that returns on stocks in January are significantly positive and larger than those in any other calendar months, which is obviously inconsistent with the EMH.

Rozeff and Kinney (1976) provide the first empirical study of the January effect for NYSE. Subsequently, Keim (1983), Roll (1983) and Haugen and Jorion (1996) document the existence of the January effect in the U.S. stock markets. Seval studies present similar monthly pattern in the non-U.S. markets, such as Brwon *et al.* (1983) in the Australia markets, Gultekin and Gultekin (1983) on seventeen industrialized countries, Tinic *et al.* (1987) in the Canadian markets, Aggarwal *et al.* (1990) in the Tokyo Stock Exchange, Barone (1990) in the Milan Stock Exchange and Ho (1990) in the six emerging Asia Pacific stock makets. In addition, some studies demonstrate different patterns of the monthly effect. For example, Berges *et al.* (1984) indicate the evidence of the July seasonality in the Canada stock markets. Fountas and Segredakis (2002) provide the findings of monthly anomalies in eighteen emerging stock markets, while very little evidence in favor of the January effect has also been found. Literature on montly patterns about agricutural futures markets is limited. Roll (1984) first finds that the price behavior of U.S. frozen concentrated orange juice (OJ) futures appears larger January returns. Thus, OJ futures seems to display monthly seasonality similar to equities. Gay and Kim (1987) also document that there is low returns in December and high returns in January for the Commodity Research Bureau (CRB) futures index. Furthermore, Liew and Brook (1998) report the significant monthly effect in four months of year (February, March, June and July) for the Kuala Lumpur crude palm oil futures.

In recent years, a persistent increase in worldwide futures trading is noticeable; especially commodity futures markets experience exceptional prices hikes and volatility after 2007 (Fung *et al.* 2010). According to the report of *Futures Industry March 2013*, the trading volume of the agricultural futures remarkably soared and

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[4]. The most common calendar anomalies documented in the literature are the day-of-the-week effect, monthly effect and holiday effect.

reached 1,270 million contracts in 2012, up 42% from 2008. Although the agricultural futures market is as important as the stock market for investors, the examination of monthly anomalies for the agricultural futures has received relatively little attention; particularly for the four historical and active futures namely corn, soybeans, soybean meal and wheat. For these four futures, there has been a rapid increase in the Chicago Board of Trade (CBOT) futures trading since 2007 and an enormous number of the 174 million contracts traded on CBOT in 2012 hit a record high, according to the statistics of *Data Stream* database.

Most of the previous studies use the common and convenient regression models, such as OLS, ARIMA and GARCH to examine the calendar effect. The shortcoming of these models is to invoke the normal return distribution assumption because Beedles (1979) and Schwert (1990) have found that individual and portfolio stock distributions have both positive and negative skewness. In fact, the normality of return assumption is inappropriate for various types of financial instrument price because they cannot drop below zero, a result at odds with the normal distribution. In addition, Kim (2006) asserts that the previously found strong January effect in the stock markets might result from the use of misspecified models in adjusting for risk.

Seyhun (1993) first uses the stochastic dominance (SD) approach to examine the monthly effects.[5] January returns of the NYSE firms are found to dominate returns of other months by first-, second-, or third-order stochastic dominance. Recently, Lean *et al.* (2007) uses the SD test of Davidson and Duclos (2000) (DD test) to examine whether the calendar effects exist in Asian stock markets. The evidence shows that the January effect has disappeared in the Asian markets.

This study first applies the Linton, Maasoumi, and Whang (2005) (hereafter LMW) test based on SD theory with and without risk-free assets to examine the monthly pattern for the four active U.S. agricultural futures, including the corn, soybeans, soybean meal and wheat.[6] The LMW test is well suited for financial time

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[5]. SD approach offers three distinct advantages: First, SD theory is distribution-free in the sense that the distribution of returns can be any type of distribution and the assumption of normality is unnecessary. Second, SD approach makes minimum assumptions about investor utility function. For example, the first-order stochastic dominance (FSD) rule assumes only that investors prefer more return to less; i.e., investor utility function can be concave, linear or convex. Third, SD theory studies the entitre distribution of returns directly; i.e., it covers all information from distribution.

[6]. Two categories of SD tests have presented in the literature. One is the minimum/maximum statistic test developed by McFadden (1989) and followed by Klecan *et al.* (1991) and Kaur *et al.* (1994). Barrett and Donald (2003) propose a Kolmogorov-Smirnov type test, and Linton *et al.* (LMW, 2005) extend their work through relaxing the *iid* assumption. The other class of SD test proposed by Anderson (1996, 2004) and Davidson and Duclos (DD, 2000) compares the underlying distributions on a set of grid points. The DD test is one of the most powerful examinations (see Lean et al., 2008), however, it requires the *iid* assumption for the observation to be compared. Importantly, the LMW

series data, such as each monthly return series in this study; because it can accommodate serial correlation in the data rather than requires the data to be independently and identically distributed. In particular, this test allows for the general dependence among the distributions which are to be compared, such as GARCH or stochastic volatility. Besides, our methodology utilizing SD theory allows part of investors' money to be invested in the agricultural futures (risky assets) and part of their money to be invested in the risk-free assets. That is, not only is the SD method an effective tool for comparing among risk alternatives, but it can also help investors in choosing their investment strategies for assets allocation.

Several interesting results of our study are noteworthy. First, the empirical results document the existence of the monthly effect for four agricultural futures in the U.S. markets during the sample period. Specifically, the dominative months show in October for corn, April for soybeans and August for soybean meal and wheat futures. Second, the simulation results further demonstrate that the higher trading profits can be earned if the detected monthly effect pattern is followed. Third, allocating part of investors' portfolio in risk-free asset is useful to help distinguish the performance among months for various futures, which implies that investors can choose an optimal ratio of investment between risky and risk-free assets with SD theory. The rest of this paper is organized as follows. Section 2 presents the data and methodology. Section 3 reports the empirical findings and the simulation results. Finally, Section 4 is the conclusions.

2. Data and Methodology

This first part of this section presents the data and the stochastic dominance theory is introduced in the second part of this section.

2.1 Data

All of the four contracts (corn, soybeans, soybean meal and wheat) are traded on the CBOT. The daily settlement prices are collected from the *Data Stream* database and the sample period is from January 1979 through December 2012 for all four futures.

We only use nearby contracts are considered for analysis. Similar to other studies, we use the standard nearby (nearest to delivery) contract during the delivery

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test is found to be efficient and permits general dependence among the distributions and non-*iid* observations.

month to mitigate the maturity effect on futures prices. To illustrate, the corn futures contracts mature in the months of March, May, July, September and December. We use the December contract to compute daily November returns, while the March contract is used to calculate returns for days in December.

The daily return of a particular contract is calculated as $r_{it} = (P_{it} - P_{it-1})/P_{it-1}$, where r_{it} represents the return on day t for contract i, and P_{it} and P_{it-1} are settlement prices on trading day t and t-1 for contract i. On the rollover day, the first day a new contract is chosen, P_{it-1} is considered to be the settlement price on the new contract for the previous day. This consideration avoids spurious price changes related to rollovers. Monthly returns are constructed by daily data, rather than by investigation of the patterns at the monthly level, to capture the most information about futures price of the month. Therefore, the compound interest method is used to calculate the monthly return, R_{it}.[7]

Table 1 gives information about the mean, median and standard deviation of monthly returns for the four futures contracts over the sample period. Some findings can be noted. First, Table 1 shows that the highest mean return appears in non-January for all four contracts. Specifically, the highest mean return shows in October for corn, April for soybeans, August for soybean meal and wheat. Thus, monthly pattern of the agricultural futures seems to display different from that of the stock market. Second, with regard to the median of monthly returns, the highest median shows in March for the corn and soybeans, August for the soybean meal and October for the wheat. Third, the volatility of corn and wheat is higher in June and July compared to other months. Similarly, the higher volatility of soybeans and soybean meal is in June, July and August.

This study also investigates the normality of monthly returns for each commodity futures contract by the J-B (Jarque-Bera) test. The result of each futures shows the distribution of monthly returns does not follow a normal distribution.[8] Therefore, we may conclude that it is appropriate to examine the monthly effect by the SD theories for these four futures.

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4. R_{it}, the return of month t for contract i, is calculated as $R_{it} = [(1+r_{i1})(1+r_{i2})...(1+r_{in})]-1$, where r_{it}, $t = 1,2,...n$ represents the return on day t for contract i and n is the last trading day of the month.

8. For space consideration, the results are omitted, but available upon request.

Table 1: Means, Medians and Standard Deviations (in Percent) for Monthly Returns for Four Futures Contracts during the Sample Period[1]

Month	Corn	Soybeans	Soybean Meal	Wheat
Jan	0.50%	-0.99	-1.36	-0.63
	(0.20%)[2]	(-0.35)	(-0.26)	(0.49)
	(5.53%)[3]	(5.25)	(5.92)	(5.26)
Feb	0.98	1.13	-0.23	-1.46
	(0.83)	(0.76)	(-1.07)	(-2.08)
	(4.98)	(7.19)	(7.10)	(6.67)
Mar	0.55	0.99	1.36	-1.50
	(1.48)	(1.53)	(1.34)	(-2.03)
	(5.91)	(6.16)	(6.38)	(7.37)
Apr	0.19	1.43	1.28	0.10
	(-0.69)	(0.38)	(1.30)	(-0.71)
	(4.85)	(4.37)	(6.61)	(7.61)
May	-0.38	-0.22	0.37	-0.75
	(-0.91)	(-0.79)	(-0.59)	(-2.01)
	(4.83)	(6.81)	(7.34)	(7.75)
Jun	-0.94	0.84	2.11	-1.08
	(-2.72)	(-0.38)	(1.25)	(-1.76)
	(12.55)	(7.86)	(8.35)	(9.58)
Jul	-2.82	-1.53	-0.56	1.15
	(-6.01)	(-1.81)	(-1.98)	(0.09)
	(11.14)	(10.08)	(10.60)	(10.00)
Aug	0.10	1.41	3.08	1.18
	(0.10)	(0.60)	(2.20)	(0.39)
	(6.83)	(8.02)	(8.43)	(6.52)
Sep	-2.18	-2.45	-2.23	-0.52
	(-2.20)	(-2.18)	(-1.66)	(0.48)
	(7.54)	(7.42)	(7.57)	(7.89)
Oct	1.39	0.72	2.35	0.12
	(0.24)	(0.25)	(-0.04)	(1.61)
	(7.42)	(6.92)	(7.49)	(7.48)
Nov	-0.28	0.75	1.07	-0.26
	(-0.78)	(0.89)	(-0.07)	(0.62)
	(6.17)	(4.76)	(6.42)	(5.63)
Dec	0.49	-0.01	0.28	-0.59
	(-0.70)	(-1.34)	(-0.38)	(-0.28)
	(6.12)	(6.09)	(7.13)	(6.51)

1. The staring date is 1979/1 and the ending date of all futures is 2012/12 for the four futures.
2. The figures in parentheses are the median of returns.
3. The figures in parentheses are the standard deviation of returns.

2.2 Stochastic Dominance

Stochastic dominance (SD) theory provides a simple approach for choosing among risky alternatives.[9] Our monthly returns data are compared using the first three SD rules; i.e. first-, second- and third-degree stochastic dominance, denoted by FSD, SSD and TSD, respectively. This study also applies first-, second- and third-degree stochastic dominance with a risk-free asset, denoted by FSDR, SSDR and TSDR, respectively, or generally called by SDR rules to distinguish from SD rules.[10]

An investor attempts to choose between two risky assets, Z_1 and Z_2. Denote by F_1 and F_2 the cumulative distributions of the return of two risky options Z_1 and Z_2, and R_f denotes the return on the risk-free assets. Let $Q_{F_1}(p)$ and $Q_{F_2}(p)$ denote the pth order quantiles of the distributions F_1 and F_2. Let U_1 be the set of all non-decreasing utility functions; U_2 the set of all non-decreasing concave utility functions; U_3 the set of all non-decreasing concave utility functions with convex marginal utility. We define that F_1 dominates F_2 in U_i, (for $i = 1, 2, 3$) or $F_1 D_i F_2$, if for every $u \in U_i$, $E_{F_1} u(x) \geq E_{F_2} u(y)$. Using the quantile approach, we write FSD, SSD and TSD rules before turning to SDR rules.

Theorem 1 (FSD): $F_1 D_1 F_2$ if and only if, $Q_{F_1}(p) \geq Q_{F_2}(p)$, $\forall p$ with a strict inequality for at least one p.

Theorem 2 (SSD): $F_1 D_2 F_2$ if and only if $\int_0^p [Q_{F_1}(t) - Q_{F_2}(t)]dt \geq 0$, $\forall p$ with a strict inequality for at least one p.

Theorem 3 (TSD): $F_1 D_3 F_2$ if and only if $\int_0^p \int_0^t [Q_{F_1}(z) - Q_{F_2}(z)]dz dt$, $\forall p$ with a strict inequality for at least one p. And, in addition,

$$\int_0^1 [Q_{F_1}(t) - Q_{F_2}(t)]dt \geq 0$$

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[9]. The stochastic dominance rules given here are slightly modified from Hadar and Russell (1969), Hanoch and Levy (1969), Levy and Kroll (1979), Seyhun (1993), Liao and Chou (1995), Levy (1998) and Best *et al*. (2000). The readers interested in SD theory should consult Levy and Kroll (1976, 1978), Kroll and Levy (1980), Levy (1992), Post (2003) and Linton *et al.* (2005).

[10]. SDR is a sharper tool than SD to establish dominance because SDR is powerful at narrowing the effieicent set. In the Appendix A, Figure A2 shows how FSDR rotates the CDF(cumulative distribution function) to help achieve dominance when FSD fails

When borrowing and lending at the risk-free rate are permitted, we turn to introduce the SDR rules. Define new random variables Y_α and Z_β as follows: $Y_\alpha = (1-\alpha)R_f + \alpha Y$ and $Z_\beta = (1-\beta)R_f + \beta Z$, where α and β are positive constants. Let $F_{1\alpha}$, and $F_{2\beta}$ denote the cumulative distribution of Y_α and Z_β, while $\{F_{1\alpha}\}$ and $\{F_{2\beta}\}$ denote the sets of distributions of all possible mixes of Y and R_f as well as Z and R_f, respectively. We say that $\{F_{1\alpha}\}$ dominates $\{F_{2\beta}\}$, if and only if for each element $F_{2\beta} \in \{F_{2\beta}\}$ there exists at least one element in $\{F_{1\alpha}\}$ which dominates it. To simplify the expression in the following theorem, define

$$\delta(p) \equiv [Q_{F_2}(p) - R_f]/[Q_{F_1}(p) - R_f] \qquad (1)$$

$$\gamma(p) \equiv \int_0^p [Q_{F_2}(t) - R_f]dt \Big/ \int_0^p [Q_{F_1}(t) - R_f]dt \qquad (2)$$

$$u(p) \equiv \int_0^p \int_0^t [Q_{F_2}(z) - R_f]dzdt \Big/ \int_0^p \int_0^t [Q_{F_1}(z) - R_f]dzdt \qquad (3)$$

Theorem 4 (FSDR, SSDR, TSDR): $\{F_{1\alpha}\}D_i\{F_{2\beta}\}$ if and only if:

(a) For $i = 1$, $\displaystyle\mathop{INF}_{0<p<F_1(R)} \delta(p) \geq \mathop{SUP}_{F_1(R)<p<1} \delta(p)$

(b) For $i=2$, $\displaystyle\mathop{INF}_{0<p<p_0} \gamma(p) \geq \mathop{SUP}_{p_0<p<1} \gamma(p)$ where p_0 is determined by the equation

$$\int_0^{p_0} \left[Q_{F_1}(t) - R_f\right]dt = 0$$

(c) For $i=3$, $\displaystyle\mathop{INF}_{0<p<p_1} u(p) \geq Max[\mathop{SUP}_{p_1<p<1} u(p), \gamma(1)]$

where P_1 is a value in the range [0, 1] that solves the equation $\int_0^{p_1} \int_0^t [Q_{F_1}(z) - R_f]dzdt = 0$; if there is no such value P_1, then the condition is simply $\displaystyle\mathop{INF}_{0<p<1} u(p) \geq \gamma(1)$.

As noted above, several tests of stochastic dominance have been proposed in the econometrics literature. This study uses a bootstrap-based test due to Linton, Maasoumi, and Whang (LMW, 2005). The LMW test is briefly introduced in the Appendix B.

3. Empirical Results

Three parts are discussed in this section. In Section 3.1, the returns of corn futures are first examined whether the monthly effect exists. And, Section 3.2 investigates the dominance relationship among the twelve months for the soybeans, soybean meal and wheat futures, respectively. The last section, Section 3.3, presents the simulation results.

3.1 Results of Corn Futures

Figure 1 shows the cumulative density functions (CDFs) of the corn futures for March, May, September, October and November during the sample period.[11] Seven other months are omitted for space consideration. On the whole, the October returns lie to the right of the other four monthly returns, implying that return in October appears to outperform the other returns in the four months. However, the two cumulative distribution curves between October and these four months, for example, cross each other in the returns range approximately between -22% and 22%.[12] As a result, there is no FSD relationship between October and these four months. The formal LMW test is subsequently used to examine dominance relationship among months' returns.
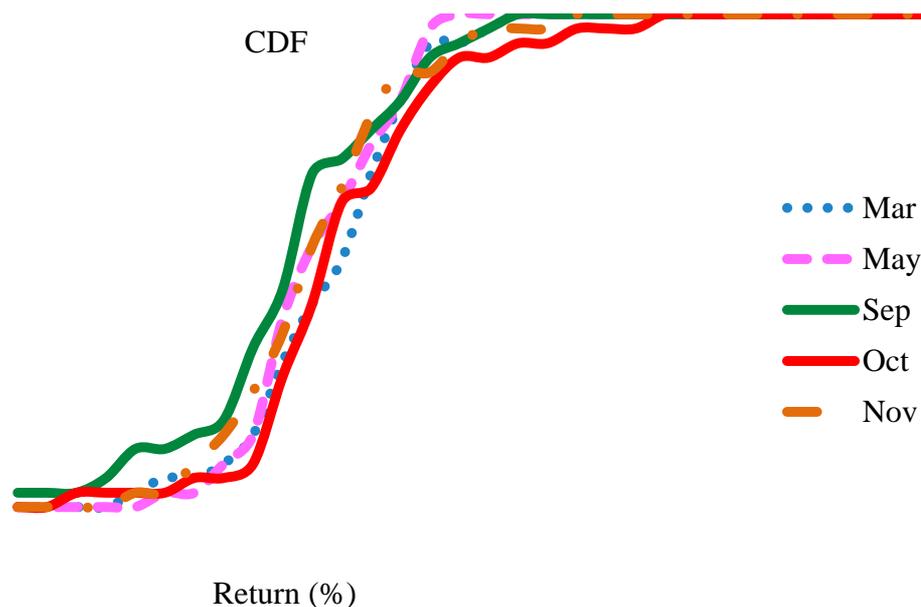

Figure 1: The CDFs of the Monthly Returns for the Corn Futures Contract for the Months of March, May, September, October and November

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[11]. Figures 2, 3 and 4 also exhibit the CDFs of some monthly returns for the soybeans, soybean meal and wheat futures during the sample period, respectively.

[12]. For the five months during the stusy period, the maximum rate of return, 22.1%, appear in September 2011 and minimum rate of return, -22.8%, is in October 2006.
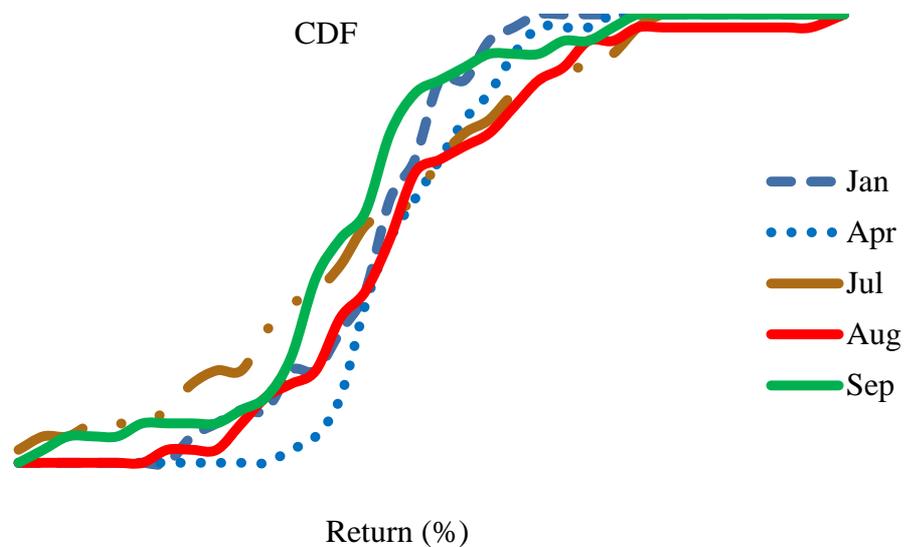
CDF

Return (%)

Figure 2: The CDFs of the Monthly Returns for the Soybeans Futures Contract for the Months of January, April, July, August and September
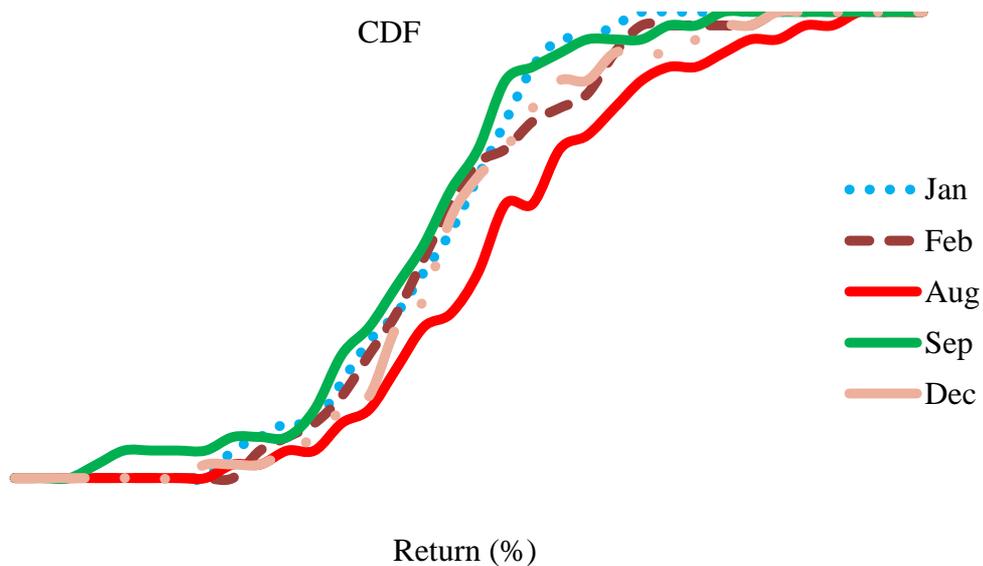
CDF

Return (%)

Figure 3: The CDFs of the Monthly Returns for the Soybean Meal Futures Contract for the Months of January, February, August, September and December
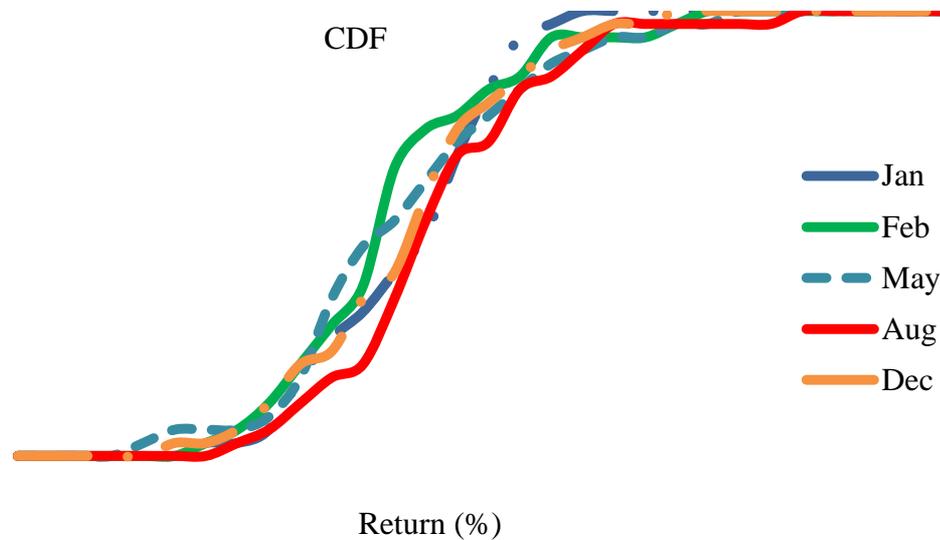
Figure 4: The CDFs of the Monthly Returns for the Wheat Futures Contract for the Months of January, February, May, August and December

Our testing strategy with LMW test is designed as follows.[13] Let X stand for a target month and Y denote individual non-target month. To establish the direction of stochastic dominance between X and Y, the LMW test is used to examine for two null hypotheses. The first null hypothesis, $H_0^1 : X \succ_S Y$, is that a target month stochastically dominates non-target month at the *sth*-degree. The second null hypothesis is the converse of the first null hypothesis, i.e., $H_0^2 : Y \succ_S X$. We conjecture that target month outperforms non-target month if we accept H_0^1 and reject H_0^2. We also conjecture that returns on target month are not excessively high if neither of the null hypotheses can be rejected. Results of the LMW test for the four futures during the sample period are shown in Table 2.[14] We first discuss for corn futures in great detail as follows.

First, applying the weakest assumption on investor preferences of a non-decreasing utility function ($U^{'}(r) \geq 0$), i.e., the FSD test can be used to examine the performance among the twelve months for the corn futures. The result, for example, shows that the p-value between October and July for H_0^1 is 0.54, above 10%.[15] In

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[13]. This study follows the Fong's (2010) idea to design our test strategy.
[14]. Using the LMW test, we respectively compare the pair-wise dominance relationship between each two months for the four futures. The number of comparison between any two months for each futures is C (12, 2) = 66, only the relationship between dominative month (winner's month) and non-dominative months is reported for space reason.
[15]. The p-value is calculated as the Equation (A-9) in the Appendix B. This study adopts 10% significance level.

contrast, the p-value of opposite hypotheses for H_0^2 is zero. The finding indicates that October outperforms July with FSD test. The evidence between October and September also appears the similar phenomena, in other words, October returns dominate September returns with FSD test. Nevertheless, the results reveal that October cannot outperform the other nine months with FSD test. Thus, sharper SSD test is required to distinguish among various months.

Second, assuming that investors are risk averse ($U^{'}(r) \geq 0$ and $U^{''}(r) \leq 0$), most economists accept. It implies that the second-order stochastic dominance (SSD) test can be used to investigate whether the target month (October) can beat the other months. The result, for instance, displays that the p-value between October and May for H_0^1 is 0.33, above 10%. In contrast, the p-value of opposite hypotheses for H_0^2 is 0.07, less than 10%. The findings demonstrate that October outperforms May with SSD test.

However, the performance between October and other individual months, except May, July and September, cannot be clearly distinguished with FSD or SSD test. Allowing the investors to borrow or lend money at a risk-free interest rate can help to distinguish the dominance between October and some non-October months with SSDR test. For example, the p-value between October and January for H_0^1 is 1.00, while the p-value of opposite hypotheses for H_0^2 is zero. The findings indicate that performance of October dominates that of January with SSDR test.[16] Also, the results between October and the other six months appear the similar phenomena, that is, October outperforms February, March, June, August, November and December with SSDR tests, respectively. So far, the returns between October and April cannot be clearly distinguished with SSD or SSDR test; therefore, the higher order SD test, i.e. TSD or TSDR, is required to compare the performance between these two months.

Third, the more powerful TSDR test, assumptions of $U^{'}(r) \geq 0$, $U^{''}(r) \leq 0$ and $U^{'''}(r) \geq 0$, is used to examine the dominance relationship between October and April for corn futures. The evidence appears that the p-value between these two months for

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[16]. October outperforms January for $R_f \geq 1.2\%$. The annually risk-free rate is equal to 1.2%; therefore, monthly risk-free rate is equal to 0.1%. The annual risk-free interest rate of this study is taken from the *Federal Reserve Bank of St. Louis*. During the study period, the minimum and maximum values of annual risk-free interest rate are 0.4% and 14.7%, and its mean and standard deviation are about 6.0% and 2.5%.

H_0^1 is 1.00, highly above 10%, while the p-value for their opposite hypotheses for H_0^2 is zero. In other words, October definitely outperforms April with TSDR test.

In short, our results show the October outperforms all other eleven months for corn contracts. Further, allocating part of investors' assets in risk-free ones is useful in distinguishing returns of October among those of non-October, which implies that investors can choose an optimal ratio of investment between risky and risk-free assets with SD theory.

Table 2: The Results of Stochastic Dominance Tests for the Four Agricultural Futures

Month	Corn (X: Oct[1], Y: Individual Non-Oct)			Soybeans (X: Apr Y: Individual non-Apr)			Soybean Meal (X: Aug, Y: Individual non-Aug)			Wheat (X: Aug, Y: Individual Non-Aug)		
	$H_0^1: X \succ_s Y$	$H_0^2: Y \succ_s X$		$H_0^1: X \succ_s Y$	$H_0^2: Y \succ_s X$		$H_0^1: X \succ_s Y$	$H_0^2: Y \succ_s X$		$H_0^1: X \succ_s Y$	$H_0^2: Y \succ_s X$	
Jan	SSDR[2]	1.00[3]	0.00	FSD	0.77	0.07	FSD	0.66	0.06	SSD	1.00	0.05
Feb	SSDR	0.34	0.03	SSDR	1.00	0.04	SSD	0.41	0.03	SSDR	1.00	0.00
Mar	SSDR	1.00	0.00	FSDR	0.20	0.01	SSDR	0.27	0.03	SSDR	1.00	0.00
Apr	TSDR	1.00	0.00	**Winner**			SSDR	0.23	0.00	SSDR	1.00	0.09
May	SSD	0.33	0.07	SSD	1.00	0.03	SSD	1.00	0.03	SSDR	1.00	0.00
Jun	SSDR	1.00	0.00	SSDR	1.00	0.01	SSDR	0.21	0.00	SSD	1.00	0.03
Jul	FSD	0.54	0.00	SSDR	1.00	0.00	SSDR	1.00	0.07	SSDR	1.00	0.00
Aug	SSDR	1.00	0.00	SSDR	1.00	0.03	**Winner**			**Winner**		
Sep	FSD	0.76	0.06	FSD	0.33	0.00	FSD	0.74	0.00	SSDR	1.00	0.06
Oct	**Winner**			SSDR	1.00	0.00	TSDR	1.00	0.09	SSDR	1.00	0.04
Nov	SSDR	1.00	0.00	SSDR	1.00	0.07	SSD	0.50	0.08	SSDR	1.00	0.00
Dec	SSDR	1.00	0.00	SSDR	1.00	0.00	SSDR	1.00	0.00	SSDR	1.00	0.00

1. The total number of comparison between any two months for corn futures is C(12, 2) = 66, only the median of corresponding *p*-value between October (winner's month) and non-October is reported for space reason.

2. FSD: first-degree stochastic dominance, SSD: second-degree stochastic dominance, TSD: second-degree stochastic dominance, FSDR: first-degree stochastic dominance with a risk-free asset, SSDR: second-degree stochastic dominance with a risk-free asset, TSDR: third-degree stochastic dominance with a risk-free asset. The portfolio return (for the mixture quantile) is calculated as $Q_a = aQ_w(p) + (1-a) r_f$. *a* is the ratio of investor money is invested in the risky assets and $(1-a)$ is the ratio of investor money is invested in the risk-free assets. Note that we use the loop method to search the proportion of $(1-a)$ for the various futures. For example, the output of our program shows that if monthly rate on returns of risk-free asset is greater than 0.1%, i.e., $r_f \geq 0.1\%$ (the annually risk-free rate, R_f, is equal to 1.2%; therefore, monthly risk-free rate is equal to 0.1% (0.012/12)) and α interval is [21%, 68%], then the mixture quantile $Q_\alpha(p)$ can dominate January returns by SSDR; i.e., October returns can dominate January returns by SSDR for $R_f \geq 1.2\%$.

3. The p-value is calculated as the Equation (A-9) in the Appendix B.

3.2 Results of Soybeans, Soybean Meal and Wheat Futures

Figure 2 shows the CDFs of monthly returns of the soybeans futures for the months of January, April, July, August and September. A visual of figure shows that the April and August returns may outperform the other three monthly returns with a certain SD or SDR test for the soybeans futures. Furthermore, Figures 3 and 4 seemingly appear that the August returns may outperform the other monthly returns for soybean meal and wheat futures, respectively. As corn futures, the results of LMW test for the other three futures during the sample period are also presented in Table 2 and briefly discussed as follows.

In Table 2, the test results of the soybeans futures reveal that returns of April, for instance, can dominant those of January with the FSD test because the p-value between these two months for H_0^1 is 0.77 and the p-value for their opposite hypotheses for H_0^2 is 0.07. Also, the result appears that April respectively outperforms September and March with FSD or FSDR test. In addition, the evidence indicates that April can beat the other eight months, including February, May, June, July, August, October, November and December, with SSD or SSDR test. These findings show that April is the stochastic dominance month for the soybeans futures.

For the soybean meal, the results indicate August outperforms January with FSD test because the p-values between these two months for H_0^1 and H_0^2 are 0.66 and 0.06, respectively. Also, the finding between August and September appears the similar phenomena, that is, returns of August dominate those of September with FSD test. Then, we discuss the results of SSD and SSDR tests. The evidence reveals that August respectively beats the other eight months, including February, March, April, May, June, July, November and December, with SSD or SSDR test. In addition, August outperforms October with TSDR test. Our findings of soybean meal appear that August outperforms the other eleven months using the SD test, consistent with a visual of the Figure 3.

For the wheat, the results of SSD test indicate that performance of August is superior to that of January and June, respectively. The finding also demonstrates that August returns dominate February returns with SSDR test. Similarly, August can beat the other eight months with SSDR test namely March, April, May, July, September, October, November and December. Therefore, the results show that August is the dominative month for the wheat futures.

In short, the empirical results present the existence of the monthly effect in the four U.S. agricultural futures markets; specifically, our findings indicate that the returns in October for corn, April for soybeans and August for soybean meal and wheat dominate returns in other months. Also, the monthly effect patterns are generally consistent with seasonal cycle in crop production.[17] For example, April for soybeans the month where there are much lower stock levels than at harvest season, average returns are dominative positive.[18]

3.3 Simulation

To examine the power of SD approach, we conduct a trading strategy to capture the return pattern. Investors are assumed to buy the amount of one million dollars for each futures contract at the settlement price of the last trading day in the month *t* and sell them at the settlement price of the last trading day in the month *t+1*. We calculate the profits or losses for the various futures every month and then sum them up during the sample period. Table 3 reports the profits/losses results with and without transaction costs.[19]

The simulation results of without transaction costs during the whole sample period show that October for the corn, April for soybeans and August for the soybean meal and wheat respectively earn the highest profits. All of these four winner's months are consistent with the results of SD tests in the Table 2. We also conduct the trading profits/losses of the simulation investment with transaction costs and the results are similar to those without transaction costs. These findings are meaningful to investors, that is, higher trading profits can be made according to the detected monthly effect patterns.

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[17]. Crop cycle-related seasonalities in agricultural commodities are documnted by Roll (1984) and Milonas (1991). Because all agricultural commodities must follow their own cyclic nature of production the stages of development from planting to harvest, which repeats the same seasonal patterns year after year. For example, the season of soybeans planting is in the summer, harvesting begins about September and lasts until November, and inventories typically increase over the winter months. After their highest level at harvest, stock levels decline as the year progress. During the following spring and summer, storage is much lower than at harvest, and low supply pushes the commodity prices during this period., e.g. March and April.

[18]. As demonstratedd in the Table 1, the highest mean return shows in April for soybean futures.

[19].Transaction costs include commission fees and transaction taxes in this study. About 0.6‰ round-trip transaction costs are considered in the U.S. market, which is estimated from Liu's (2005) paper.

Table 3: Trading Profits/Losses of the Simulation Investment with and without Transaction Costs during the Sample Period [1]

Month/Futures	Corn	Soybeans	Soybean Meal	Wheat
Jan	$170364 [2]	-336814	-461955	-214019
	(149964)	(-357214)	(-482355)	(-234419)
Feb	331853	385468	-79527	-496344
	(311453)	(365168)	(-99927)	(-516744)
Mar	186267	337736	462367	-511228
	(165867)	(317336)	(441967)	(-531628)
Apr	65267	481787 [a]	436742	34304
	(44867)	(461591)	(416342)	(13904)
May	-128412	-74085	124149	-254335
	(-148812)	(-94485)	(103749)	(-274735)
Jun	-317967	286790	719032	-366167
	(-338367)	(266390)	(698632)	(-386567)
Jul	-958358	-521299	-191355	390565
	(-978758)	(-541699)	(-211755)	(370165)
Aug	35410	479225	945189 [a]	396259 [a]
	(15010)	(458858)	(926829)	(376063)
Sep	-740270	-833476	-757798	-176432
	(-760670)	(-853876)	(-778198)	(-196832)
Oct	342410 [a3]	244202	797389	39577
	(363755)	(223802)	(776989)	(19177)
Nov	-96336	255811	363180	-89855
	(-116736)	(235411)	(342780)	(-110255)
Dec	167873	-2379	96628	-201172
	(147473)	(-22779)	(76228)	(-221572)

1. According to the results in the Table 2, our trading strategy, of course, is that part of investors' money is invested in risky assets (buying futures) while part of their money is invested in risk-free assets for the dominative months.
2. The figures are profits/losses without transaction costs. The figures in parentheses are adjusted with transaction costs.
3. "a" indicates that it gains the most profit among the months and is the "winner" in the Table 2.

4. Conclusions

Many empirical studies document the existence of monthly anomalies in stock markets. Literature on monthly patterns about agricultural futures market is limited, although the agricultural futures market is as important as the stock market for investors. Therefore, this study uses the stochastic dominance theory to examine monthly effect for the four active U.S. agricultural futures namely corn, soybeans, soybean meal and wheat.

Our findings show the existence of the monthly effect, but different patterns are present across the four contracts during the sample period. Specifically, the results show that returns of October dominate those of non-October for the corn futures. The returns in

April outperform the other eleven months for the soybeans futures. In addition, return in August is superior to that of non-August for soybean meal and wheat futures, respectively.

Furthermore, the simulation results with or without transaction costs show that the dominative month for each futures can earn the higher profits than the other non-dominative months. It is worth to mention that allocating part of investors' assets in risk-free ones is useful in distinguishing returns among months for the various futures.

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Appendix A: Rotation of Cumulated Distribution Function

An investor attempts to choose between two risky assets, Z_1 and Z_2. Assume that the return on an asset Z_1 always exceeds that on asset Z_2. In this case, since investors prefer more return to less, no investor would choose the asset Z_2. Generally, the asset Z_1 dominates the asset Z_2 by FSD, if the cumulative density function (CDF) of Z_1 lies to the right of the CDF of Z_2.

In the following, Figure A1 shows that an asset Z_1 with CDF F_1 dominates an asset Z_2 with CDF F_2 by FSD. F_1 lies far enough to the right of F_2 that the asset Z_1 is preferred to the asset Z_2, because the expected utility gain from the positive area to the left of R_1 exceeds the reduction in the expected utility loss between R_1 and R_2. This example is a special case of the first-order stochastic dominance (FSD).

When two CDFs cross, we can use stronger rules, called the stochastic dominance with the risk-free asset rules (SDR) to differentiate returns. Consider a portfolio of one risky asset and one risk-free, with $\lambda\%$ of the investor's money invested in the risky asset Z_1 and $(1-\lambda\%)$ borrowed or lent at the risk-free; the portfolio return, R_p is then computed as the weighted sum of two assets: $R_p = (1-\lambda)R_f + \lambda Z_1$, where R_f is the risk-free rate.

Let F_λ be the cumulated distribution function of R_p. Figure 2 compares the two distributions F_1 and F_2. Clearly, neither F_1 nor F_2 dominates each other by FSD. Nevertheless, it is possible to rotate F_1 about the point $(R_f, F_1(R_f))$ and obtain $F_{1\lambda}$, which dominates F_2 by FSD; therefore F_1 dominates F_2 by first-order stochastic dominance with risk-free rate (FSDR). In Figure A2, rotating F_1 about the point $(R_f, F(R_f))$, we obtain F_{1a} which dominates F_2 by FSD; hence F_1 dominates F_2 by FSDR.
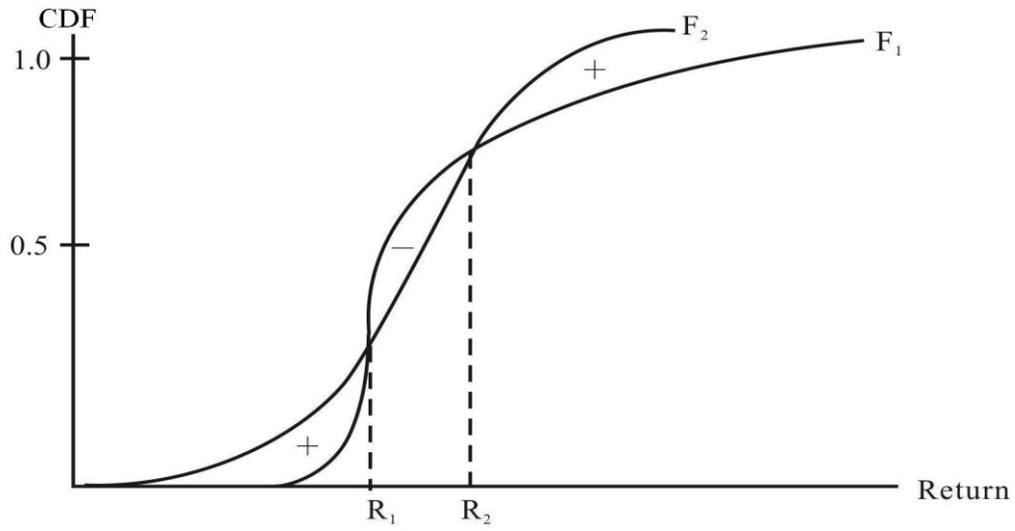
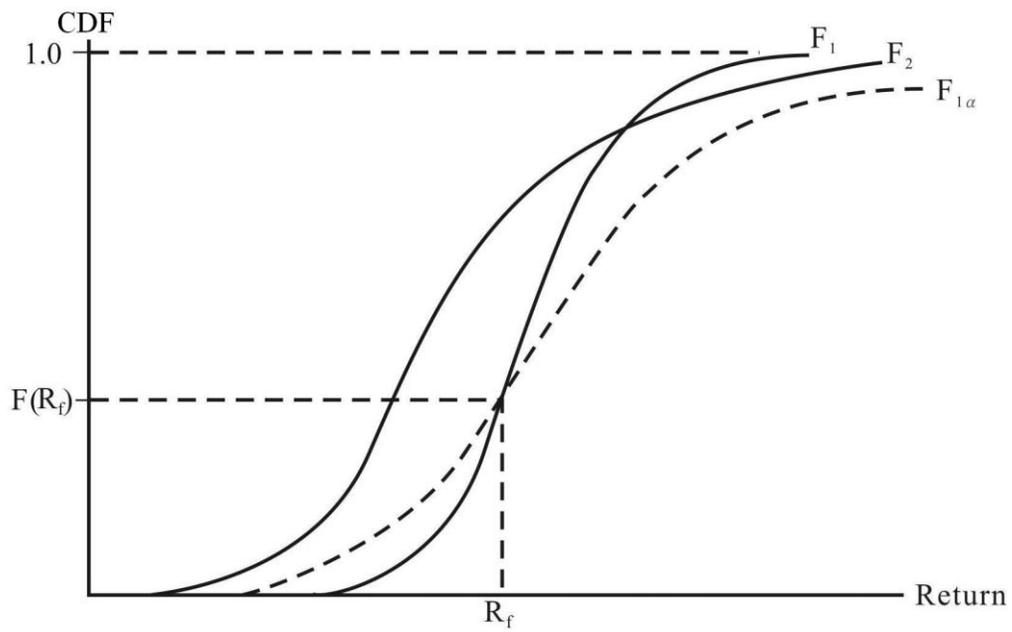Figure A1 F_1 is Preferred to F_2 with Risk Aversion

Figure A2 F_1 and F_2 intersect but $F_{1\alpha}$ Dominates F_2

Appendix B: Stochastic Dominance Test

Let $\{X_{1i}\}$, $i = 1, 2, \ldots, N$ be a random sample drawn from a population with cumulative distribution function, $F_{X_1}(r)$ and r is the uncertain return. Let $D_{X_1}^s(r)$ denote the function that integrates F_{X_1} to degree s-*1*. That is,

$$D_{X_1}^1(r) = F_{X_1}(r) \tag{A-1}$$

$$D_{X_1}^2(r) = \int_{-\infty}^{r} F_{X_1}(t)dt = \int_{-\infty}^{r} D_{X_1}^1(t)dt \tag{A-2}$$

and

$$D_{X_1}^3(r) = \int_{-\infty}^{r} \int_{-\infty}^{u} F_{X_1}(t)dudt = \int_{-\infty}^{r} D_{X_1}^2(t)dt \tag{A-3}$$

where $r > 0$ for all cases. Let $\{X_{2i}\}$, $i = 1, 2, \ldots, N$ be a random sample from a population with CDF $F_{X_2}(r)$. Define $D_{X_2}^s(r)$ analogously. The LMW test examines the following hypotheses:

$$H_0: \ D_{X_1}^s(r) \le D_{X_2}^s(r) \ \text{ for all } \ r \in V \ \text{ and}$$

$$H_1: \ D_{X_1}^s(r) > D_{X_2}^s(r) \ \text{ for some } \ r \in V \tag{A-4}$$

where *V* is the usual joint support for the two distributions.

Suppose the null hypothesis is that X_1 dominates X_2 at the *sth* degree. The alternative hypothesis is that stochastic dominance fails at some point. Two null hypotheses need to be examined to establish the direction of SD. The first null hypothesis is $H_0^1: X_1 \succ_s X_2$ where "$\succ_s$" indicate stochastic dominance at the *sth* degree. The second null hypothesis is the converse, i.e., $H_0^2: X_2 \succ_s X_1$. We infer that X_1 dominates X_2 if we accept H_0^1 and reject H_0^2. To test the two null hypotheses, the following test statistic is proposed by LMW:

$$\hat{L}^{(s)} = \sup_r \sqrt{N} \left\{ \hat{D}_{X_1}^{(s)}(r) \le \hat{D}_{X_2}^{(s)}(r) \right\} \tag{A-5}$$

where

$$\hat{D}_R^{(s)}(r) = \frac{1}{N(s-1)!} \sum_{i=1}^{N} (r - R_i)^{s-1} I(R_i \le r), \quad R = X_1, \ X_2. \tag{A-6}$$

where *N* is the sample size and $I(\cdot)$ is the indicator function. LMW show that the asymptotic null distribution of this statistic is non-standard and proposes using sub-sampling bootstrap simulations to compute the empirical p-values of the test. The idea of

the sub-sampling bootstrap procedure is to sampled blocks of data without replacement to account for non *iid* features of the data. Politis and Romano (1994) have proven that the sub-sample bootstrap consistently estimates the distribution of a statistic under very weak conditions. In the case of the LMW test, the sub-sampling method requires computing *N-b+1* times the following test statistic for a sub-sample of size *b*:

$$\hat{L}_k^{(s)} = \sup_r \sqrt{b} \left\{ \hat{D}_{X_1,k}^{(s)}(r) \le \hat{D}_{X_2,k}^{(s)}(r) \right\} \text{ for } k = 1,..,N-b+1 \qquad (A\text{-}7)$$

where

$$\hat{D}_{R,k}^{(s)}(r) = \frac{1}{b(s-1)!} \sum_{i=1}^{b} (r - R_{k+i-1})^{S-1} I(R_{k+i-1} \le r), \quad R = X_1, X_2. \qquad (A\text{-}8)$$

The empirical $\hat{p}$-value

$$\hat{p} = \frac{1}{N-b+1} \sum_{k=1}^{N-b+1} I(\hat{L}_k^{(s)} - \hat{L}^{(s)} > 0) \qquad (A\text{-}9)$$

We reject the null hypothesis at a significance level if $\hat{p} < \alpha$.