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Empirical Evidence on the Causality among Yield Curve Factors and Macroeconomic Determinants

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Abstract: This paper explores the relationship between macroeconomic determinants and yield curve factors based on the Nelson and Siegel (1987) model. We also examine the effect of feedback relationship and model predictability on the aforementioned variables. More specifically, we assume that the time-decay parameter (τ) is time-varying and use the VAR (vector auto-regression) model to investigate the relationship between four yield curve factors (level parameter, β_0; slope parameter, β_1; curvature parameter, β_2; and time-decay parameter, τ) and three macroeconomic determinants (consumer price inflation rate, bank discount rate, and inventory rate). In addition, we use the variance decomposition and impulse response analysis to examine the dynamic interactions of all above variables. The results reveal that, during the observation period, the time-decay parameter (τ) exhibits a major contributing factor to macroeconomic determinants and cannot be ignored. Moreover, we find the strong evidence of the effects of yield curve on future movements in macroeconomic determinants and evidence for a reverse influence as well. Finally, from the predictability analysis on the yield curve factors and macroeconomic determinants, we conclude that the predictions of yield curve factors, bank discount rate, and consumer price inflation can be more accurate when previous macroeconomic determinants and yield curve factors information are taken into account. This conclusion provides related institutions relatively rich information in establishing their financial policy and investment strategy.

Keywords: Macroeconomic determinants, Yield curve factors, VAR model

1. Introduction

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The term structure of interest rates (TSIR), which is also known as the spot rate curve, derives from the zero-coupon government bonds under a given default risk. How to accurately estimate a smooth yield curve is always regarded as an important empirical issue by academics and market practitioners. There are substantial literatures over several decades in using statistical technology to fit the yield curve, such as McCulloch (1971, 1975), Schaefer (1981), Vasicek and Fong (1982), Steeley (1991), and Lin (1999, 2002). Most of the large financial institutions have used the TSIR to price and hedge their positions of interest-rate-dependent securities. Moreover, the shape of the yield curve provides a good predictive indicator of future economic activity, with consequent implications for the estimations of GDP (Gross Domestic Product) and the inflation rates (Estrella and Mishkin, 1998; Estrella and Hardouvelis, 1991). Therefore, investors can make good judgments on future impact of financial and economic events through observing the changes of the yield curve, thus adjust their investment, hedging strategies, and financial positions.

Recently, the empirical studies related to yield curve have gradually focused on the prediction and application of the information embedded within the yield curve. It is quite different from traditional topics in the comparison of fitting ability. One of the most interesting topics is to explore the joint behavior of macroeconomic variables and yield curve factors because of its importance in the pricing of fixed income securities, investment strategies and economic policy. For the interaction between the interest rate level and macro economy, the pioneering researches are mostly concerned about the theoretical relationship between the short-term interest rate and macro economy. However, there is a little discussion on more generalized relationship between the macroeconomic determinants and the whole yield curve changes, which the long-term and short-term interest rates are both included.

As mentioned in the earlier paper, the majority of researches have concentrated on exploring the correlation between economic variables and short-term interest rate, rather than on the whole yield curve factors, such as Fuhrer and Moore (1995). In recent years, some finance and economy researchers have started examining what the important macroeconomic determinants to the yield curve are. Wu (2001) examined the interaction between the unexpected monetary policy and the change of the slope factor of the yield curve in the United States since 1982. His results supported that unexpected monetary policy has a significant correlation with the slope factor. Using different approaches, both Evans and Marshall (2001) and Ang and Piazzesi (2003) showed that the movement of the yield curve was not only determined by level, slope and curvature, but also by the impact of the inflation rate and the real activity. Furthermore, Wu (2003) also argued that the economical shocks between the short-term interest rate and the macroeconomic variables could spill over to the middle-term and long-term interest rate through the short-term forward rate.

In addition, the analysis of the relationship between macroeconomic determinants and the yield curve factors can be distributed into two categories, that are a unidirectional effect and a bidirectional effect. Most of the previous empirical studies (such as Wu (2001), Ang and Piazzesi (2003), and Hördahl, et al. (2006)) only considered a unidirectional effect from the real output and inflation to yield curve factors. Otherwise, Estrella and Hardouvelis (1991) and Estrella and Mishkin (1998) solely focused on the different unidirectional predictability of the yield curve factors for the economical variables based on the assumption of the yield curve to the economy linkage. Unlike these unidirectional assumptions, Diebold, et al. (2006) examined the bidirectional relationship between the economical variables and the yield curve factors without analyzing out-of sample forecasts.

However, the aforementioned literatures (unidirectional or bidirectional frameworks) all used zero coupon bond yields to estimate spot rate curves by the variant estimation method. In fact, there are relatively few emerging countries having a zero coupon bond market. In order to solve the problem of insufficient zero coupon bond samples in emerging countries, this paper aims to estimate and analyze the term structure of interest rates in the Taiwan government bond (TGB) market based on the parsimonious function specified by Nelson and Siegel (1987), i.e. we can estimate the level, slope and curvature factors embedded in this model. Then, we incorporate the yield factors into the VAR model with macroeconomic determinants, and examine the effect of feedback relationship and model predictability via impulse responses, variance decomposition techniques, and out-of-sample performance of VAR model.

Compared to other developed countries, the Taiwan government bond market has a noticeably smaller trading volume and is not so liquid. In 2010, the trading volume of the bond's secondary market reached NT$ 106 trillion[1], showing that the

[1] The average exchange rate is US$1= NT$ 31.642 in this year.

Taiwan bond market has gradually expanded. Recently, to accelerate the pace of liberalization and internationalization, the authorities have greatly eased the regulations and thus improved the trade efficiency in the secondary market. In order to attract more foreign interest and further develop Taiwan as an Asian-Pacific regional financial center, the Taiwan Futures Exchange (TAIFEX) launched its operation in July of 1998 and introduced a local 10-year Government Bond Futures for hedging and speculation purposes. After the strenuous efforts of several years, the Taiwan financial market has been placed on the top of the list of fast-growing emerging markets in the world.

The remainder of this paper is organised as follows. The next section introduces the estimation of yield-curve factors and the VAR model. The third section is our empirical results: (1) the detailed data; (2) the estimation of the VAR model of yield curve factors and macroeconomic determinants; (3) the in-sample performance of impulse responses and variance decomposition techniques; and (4) out-of-sample performance of the VAR model. In the final section, the conclusions are presented.

2. Methodology

2.1 Choosing a model of yield curve

The method adopted in this paper to fit the yield curve in the Taiwan government bond market is a parsimonious model proposed by Nelson and Siegel (1987). For analyzing the sensitivity of a fixed income portfolio to the yield-curve's level, slope, and curvature, Willner (1996) contended that the Nelson and Siegel (1987) model is a useful and powerful method. Using the same model for estimation, Dolan (1999) also pointed out that future level, slope, and curvature of the yield curve can be predicted. In a similar vein, Diebold and Li (2006) and Diebold et al. (2006) also mentioned that the well-known Nelson-Siegel (1987) model is good at extracting yield-curve dynamics and obtaining good predictions. Hence, Nelson and Siegel (1987) model is employed in our study to estimate the factors representing the level, slope, and curvature of the yield curve.

By integrating the process of the forward curve, Nelson and Siegel (1987) expressed the spot rate curve as follows:

$$R(t) = \beta_0 + \beta_1\left(\frac{\tau}{t}\right)\left[1 - exp\left(\frac{-t}{\tau}\right)\right] + \beta_2\left(\frac{\tau}{t}\right)\left[1 - exp\left(\frac{-t}{\tau}\right)\left(\frac{t}{\tau}+1\right)\right] \tag{1}$$

In the Nelson and Siegel (1987), the explanation of the four parameters are illustrated as follows: (1) the value of β_0, regarded as a long-term interest rate, representing the level of the yield curve; (2) the value of β_1, regarded as a short-term interest rate, representing the slope of the yield curve; (3) the value of β_2, regarded as a medium-term interest rate, representing the curvature of the yield curve; and (4) the parameter τ, which governs the exponential decay rate at which the short-term and medium-term factors decay to zero.

If there are no zero coupon bonds, the parameters in Equation (1) are impossible to be estimated directly using the ordinary least square regression method. However, in most of the emerging bond markets, only coupon bond issues are available. The only way to estimate parameters in Equation (1) is to make use of the coupon bond yields. Since the theoretical price of a coupon bond is equal to the sum of the present value of the future coupon and the principal payments, thus can be expressed as following:

$$\hat{B}_i = \sum_{j=1}^{D_i} C(t_{i,j})\, exp\left\{-t_{i,j} R(t_{i,j})\right\} \tag{2}$$

Where $\hat{B}_i$ is the i_{th} theoretical price of coupon bond; D_i is the maturity of the i_{th} bond; $C(t_{i,j})$ is the cash flow of the i_{th} bond at time t_j; and $R(t_{i,j})$ is the spot rate at time t_j in the i_{th} bond.

To generate these parameters of the yield curve, the function of spot rate curve is substituted to Equation (2) as follows:

$$\begin{aligned} \hat{B}_i &= \sum_{j=1}^{D_i} C\left(t_{i,j}\right)\exp\left(\left(-t\right)\left(R\left(t\right)\right)\right) \\ &= \sum_{j=1}^{z_i} C\left(t_{i.j}\right)\exp\left(\left(-t_{i,j}\right)\times\left(\beta_0+\beta_1\left(\frac{\tau}{t_{i,j}}\right)\left[1-\exp\left(\frac{-t_{i,j}}{\tau}\right)\right]\right.\right. \\ &\left.\left.+\beta_2\left(\frac{\tau}{t_{i,j}}\right)\left[1-\exp\left(\frac{-t_{i,j}}{\tau}\right)\left(\frac{t_{i,j}}{\tau}+1\right)\right]\right]\right)\right) \end{aligned} \tag{3}$$

The parameters can then be estimated by minimising the difference between the actual and theoretical bond price; that is:

$$Q = \frac{1}{n}\sum_{i=1}^{n}\left[(B_i - \hat{B}_i)\right]^2 \tag{4}$$

where n is the number of bonds.

Because the object function is nonlinear, the Newton numerical method is used to estimate the parameters of the Nelson and Siegel (1987) model. One advantage of this method is that τ can sustain its variance with other parameters. In this regard, it should be noted that Diebold and Li (2006) estimated the Nelson and Siegel (1987) model with a constant τ, but Hurn et al. (2005) argued that the curve from the Nelson and Siegel (1987) model is sensitive to the scale parameter τ, which cannot be fixed. Thus, based on the monthly data, we can generate the time series data of the estimated parameters embedded in Nelson and Siegel (1987) model, and regard them as the yield curve factors in the following VAR model.

2.2 Formulation of VAR model

Given the pioneering works of Dolan (1999) and Diebold and Li (2006), the level, slope, curvature, and τ factors can be regarded as time-varying parameters and present good proxy factors for the changes of yield curve. It is interesting to explore the feedback relationship between macroeconomic determinants and yield curve factors. According to the suggestion of Diebold, et al. (2006), the macroeconomic determinants are defined as the consumer price index growth rate (CPIG), central bank discount rate (CBDR), and Inventory Rate (IR). These variables are widely used in macroeconomic analysis and characterize the macroeconomic fundamentals, such as the inflation rate, monetary policy instrument, and level of real economic activity.

To analyze the feedback relationship between the macroeconomic determinants and the yield curve factors without prior and specific causality, we employed the VAR model by Sim (1980) to formulate these seven time series as the following VAR model:

$$F_t = A_0 + A(L)F_t + e_t \tag{5}$$

where $F'_t = \left(\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \tau_t, CPIG_t, CBDR_t, IR_t\right)$, and is a 7×1 matrix, which includes four yield curve factors and three macroeconomic determinants. The lag operator of $A(L)$ can be expressed as the polynomial function (6):

$$A(L) = A_1 L + A_2 L^2 + A_3 L^3 + \cdots + A_P L^P \quad (6)$$

where p indicates the lag periods, and A_i is a 7×7 matrix of coefficients which interprets the interaction among these variables with different lag periods. The errors e_t are orthogonal and follow a white noise process.

For the sake of lacking the assumption of the prior causality, the sequence of the variables in the VAR model would affect the consequence of the variance decomposition and impulse response analysis. Therefore, the Granger causality test should firstly used for determining the best order among the variables in the VAR model. After that, the process of variance decomposition is employed to measure the effects of the macroeconomic variables and yield curve factors to the forecast variances. Next, through the impulse response functions, we observe the behaviors of the variables in response to the various shocks from other variables. Finally, we present the predictability robustness of the VAR model with out-of-sample data.

3. Empirical Analysis

3.1 Data

The series of macroeconomic data are the growth rate of consumer price index, central bank discount rate, and Inventory Index. The series of the CPIG index are measured by taking the natural logarithm of the monthly change of the CPI from the database of the Taiwan Economic Journal (TEJ). The monthly series of the CBDR are also obtained from the TEJ. The last series are the inventory rate, derived from the Council for Economic Planning and Development, Executive Yuan, ROC, and are measured as the natural logarithm of the monthly change. Each serial data has consisted of 180 observations between January 1996 and December 2010.

The bonds chosen in our study are all coupon bonds with the maturity less than 30 years. There total 61 coupon bonds with various coupons and maturities in our sample set. They are obtained from the TEJ and Gre Tai Securities Market (GTSM). In order to incorporate the series of the macroeconomic determinants with three yield curve factors, we estimate the monthly yield curves for the Taiwan government bonds based on the monthly data from January 1996 to December 2010 (with a total of 180 months).

This paper uses the Nelson and Siegel (1987) model to estimate the yield curve in an emerging market economy, Taiwan. Figure 1 gives the depiction of the yield curves over the sample period. We observe that there are three major shapes of the yield curves in Taiwan's government bond market, and the frequency of each shape is displayed in Table 1. From Table 1, we conclude that the Taiwan government bond market mostly has a humped or upward sloping pattern of the yield curves in our observation periods.

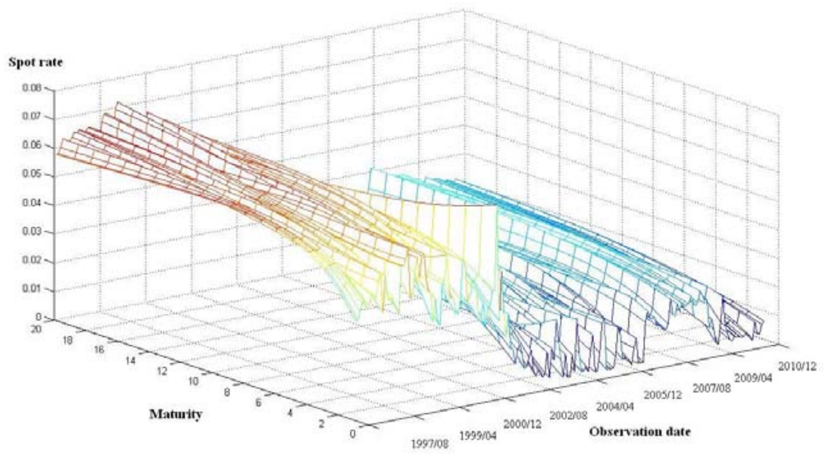

Figure 1 The time paths of estimated yield curves for Nelson-Siegel Model from January 1996 to December 2010

Table 1.
The variant shapes of the yield curves over the observation period

The shapes of the spot rate curve	Downward sloping ($\beta_1 > 0, \beta_2 > 0$)	Inverted humped ($\beta_1 > 0, \beta_2 < 0$)	Humped ($\beta_1 < 0, \beta_2 > 0$)	Upward sloping ($\beta_1 < 0, \beta_2 < 0$)
Frequency of existed pattern	1	0	138	41

From Figure 1, the varied shape of the yield curve is dominated by the changes of the yield curve parameters. Table 2 provides some descriptive statistics of both time series curve parameters and macroeconomic determinants. The left column of Table 2 shows the means, mediums, maximums, minimums, standard deviations and other statistics. Among the yield curve factors, we find : (1) the mean of the monthly β_0 is 0.0409, which governs the level change of the spot rate curve and shows that the long-term interest rate level is tended to 4.09%; (2) the mean of the monthly β_1 is -0.0248, which indicates that the yield curve is positively upward on average; (3)

the mean of the monthly β_2 is 0.0163, which shows that the slope of the yield curve is not only positive, but also has a hump shape; (4) The standard deviation of monthly τ is 5.3823, and the maximum and minimum values are 34.8662 and 0.0065, which illustrates that the range of τ is wide and should not be assumed as a constant variable.

In addition, the values of β_2 can be positive or negative, which means that the shapes of the spot rate curves in the Taiwan government bond market indeed have different patterns. The risk of the yield-curve changes should, therefore, be taken into account for effectively managing the interest rate risk.

Table 2.
Descriptive statistics of the yield curve factors and macroeconomic determinants

	β_0	β_1	β_2	τ	CPIG	CBDR	IR
Mean	0.0409	-0.0248	0.0163	5.3823	0.0009	0.0309	0.6706
Median	0.0349	-0.0292	0.0256	4.9968	0.0009	0.0275	0.6572
Maximum	0.1938	0.0203	0.1037	34.8662	0.0373	0.0550	1.0366
Minimum	0.0056	-0.0498	-0.5117	0.0065	-0.0193	0.0125	0.5282
Std. Deviation	0.0238	0.0101	0.0581	3.5885	0.0087	0.0149	0.0850
Skewness	1.9537	0.7614	-4.6109	3.6978	0.1539	0.2281	1.3871
Kurtosis	11.2432	4.3338	39.9138	28.7103	4.0187	1.0000	5.5106
Jarque-Bera	14.7381*** (0.0060)	8.4941** (0.0217)	19.1235*** (0.0030)	104.9930*** (0.0010)	624.1326*** (0.0010)	10857.553*** (0.0010)	30.7337*** (0.0010)

Note: 1. CPIG is defined as the consumer price index growth rate; CBDR is defined as the central bank discount rate and IR is defined as the inventory rate.
2. *,**,*** denote rejection of the null hypothesis of a unit root at the 10%, 5%,and 1%. The optimal lag periods are shown in parentheses.

Table 2 also shows the statistics of the three macroeconomic determinants. The mean of CPI growth rate is 0.0009, which indicates that the CPI has a slight increase during our sample period. Other variables are the monthly discount rate and monthly inventory rate. We find their means are 3.09% and 0.6706, respectively. Besides that, the maximum and minimum values of the CPI growth rate are 0.0373 and -0.0193, which shows that Taiwan's bond market faced a fluctuated change of CPI. For the central bank discount rate variable, the maximum and minimum values are 5.5% and 1.25%. This means the interest rate is effectively controlled by the Taiwan Central Bank authority. The minimum inventory rate is 0.5282 which indicates that the inventory rate could be sustained above 52.82 percent. Additionally, Table 2 shows evidence that the whole sample series are against normality in term of skewness, kurtosis, and a summary of Jarque-Bera statistics shows all statistics are statistically significant at the 10% level and reject the assumption of a normal distribution.

For the intention of achieving a more completed profile about the interrelationship between yield curve factors and macroeconomic determinants, a correlation matrix is presented in Table 3. The positive correlation between the CBDR and CPIG has confirmed that the central bank would raise the discount rate to ease the problem of inflation. And, the positive relationship between the CBDR and IR infers that a higher market interest rate would slump the business market, decrease the real output, and thus increase the inventory rate. However, all the coefficients of correlation between yield curve factors and macroeconomic determinants are not significantly high. Finally, we find the coefficient of correlation between IR and β_0 is 0.6112, which illustrates the main bridge of the yield curve factors and the macroeconomic aggregates could be constructed under this interrelationship.

Table 3.
Correlation matrix between the yield curve factors and macroeconomic variables

Variable	β_0	β_1	β_2	CPIG	CBDR	IR
β_0	1.0000	-0.5468	-0.1686	0.1339	-0.0315	0.6112
β_1	-	1.0000	-0.0646	-0.1751	0.0091	-0.1477
β_2	-	-	1.0000	0.2203	0.0654	0.0601
CPIG	-	-	-	1.0000	0.1119	-0.0662
CBDR	-	-	-	-	1.0000	0.0349
IR	-	-	-	-	-	1.0000

Note: CPIG is defined as the consumer price index growth rate; CBDR is defined as the central bank discount rate; and. IR is defined as the inventory rate.

3.2 Empirical results of VAR model

The VAR model is considerably suitable for the analysis of the stationary data because most financial time series are non-stationary. The Augmented Dickey-Fuller (ADF) should be employed here to examine whether these samples are stationary and to avoid the spurious regression problem.

The results of the unit root test for the series of original level and first lag difference are shown in Table 4. At first, among the variables without taking the difference, only β_2, τ and CPIG have significantly rejected the null hypothesis of a unit root, and are stationary series. However, the series of all variables as shown in the second column, which have taken the first difference, presented stationary at 1% significant level. Therefore, the differentiated variables, which are regarded as the change in series, are applied into the VAR model.

Table 4.
Unit root test of the yield curve factors and macroeconomic variables

	β_0	β_1	β_2	τ	CPIG	CBDR	IR
Original level	-2.9163 (2)	-1.6044 (4)	-8.5064*** (0)	-4.8097*** (2)	-16.7201*** (0)	-4.3060 (4)	1.4040 (4)
First difference level	-12.1351*** (2)	-11.1931*** (3)	-13.3390*** (1)	-16.9861*** (1)	-9.6464*** (10)	-21.0045*** (3)	-4.7323*** (3)

Note: This table reports the results of an ADF test statistic of each NS parameter. *** denote rejection of the null hypothesis of a unit root at the 1%. The optimal lag periods are shown in parentheses.

Before estimating the VAR model, an appropriate lag of sequences should be specified. The Akaike information criterion (AIC) is employed to determine the best fitting model. Through examining the residuals from the VAR model based on the tests of AIC, the optimal lag length is set at VAR (5) as shown in Table 6. It concludes that the VAR model, which is shown in Appendix 1, could capture the significant inter-reaction effect between the yield curve factors and macroeconomic determinants over the previous five months.

3.3 Granger causality test

In order to conduct the robust analysis of variance decomposition and impulse response, the Granger causality test is employed to rank the sequence of the variables in the VAR model. As shown in Appendix 2, a significant lead relationship exists between the yield curve factor τ and β_0、β_1. In addition, Appendix 2 also displays the results of the macroeconomic determinants which are dealt with unidirectional causality. The CBDR has a significantly unidirectional causality with CPIG and IR. Finally, the causality between yield curve factors and macroeconomic determinants has suggested that the direction of causality is from CPIG to β_1, and from CBDR to

β_0. Therefore, according to the above Granger causality test, we thus conclude that β_0, β_1, β_2, τ, CPIG, CBDR and IR are employed in the VAR model in order.

Table 5.
Estimation of the lag period test of the VAR

VAR (*P*) *	Determinant RC	c	AIC
VAR (10) *	2.40E-23	60	-30.1566
VAR (9) *	2.51E-23	54	-30.2164
VAR (8) *	2.65E-23	48	-30.2964
VAR (7) *	4.26E-23	42	-29.9870
VAR (6) *	3.96E-23	36	-30.2453
VAR (5) *	3.90E-23	30	-30.4650

Note：
1. The alphabet *P in* parentheses denotes the lag period.
2. Determinant RC is a determinant of the error covariance matrix.
3. The alphabet c presents the degree of freedom of the χ^2 distribution.
4. We also use the criterion of AIC to choice the best model.

3.4 Analysis of the decomposition of forecast error variance

The decomposition of forecast error variance regarding the yield curve factor and macroeconomic determinant variations for the seven series are illustrated in Table 6. The main illustrations of the relationship between the yield curve factors and macroeconomic determinants are as follows: (1) Both the changes of yield curve factors and macroeconomic determinants are mainly dominated by themselves, their relative yield curve factors or macroeconomic variables; (2) However, τ obviously has a heavy impact from the yield curve factor β_0; (3) The volatility for β_0 could be explained largely at 2.0104% by CPIG than by other macroeconomic variables, except for other yield curve factors. So, the changes of the inflation rate could give better explanations for the changes of the long-term interest rate; (4) As for the volatility of β_0, the short-term and mid-term interest rate, β_1 and β_2 are more heavily affected by the macroeconomic determinants, i.e. CPIG and IR. This result has indicated that the changes of the CPIG and IR have effects on the short-term and middle-term interest rate; (5) The changes of CPIG could be largely influenced by β_2 except for other macroeconomic determinants, while the changes of the IR are largely affected by β_1; (6) The CBDR does not have a large impact on any yield curve factor, whereas it is largely affected at 6.6647% and 2.1985% by the yield curve factors, i.e. β_1 and β_2.

The results of the feedback relationship in Table 6, indicate that only the short-term and mid-term yield curve factors have a significant feedback relationship with the macroeconomic determinants, GCPI and IR; while the macroeconomic determinant, CBDR, has largely been influenced by β_1 and β_2 with uni-direction. However, the long term interest rate factor is less relevant to the macroeconomic determinant.

3.5 Analysis of the impulse response function

In order to obtain a better comprehension on the shock effect between the yield curve factors and macroeconomic determinants, the impulse response analysis among seven time series is examined by employing the VAR model as shown in Figures 2 to 8. The results of the impulse function have presented the evolution of a standard deviation unit shock for the seven factors or variables individually, and are illustrated as follows. The standard deviation unit shocks of other yield curve factors have long-lasting effects on the macroeconomic determinants. Even six months since the shock occurring, noticeable oscillations still could be detected. In other word, the shocks to the macroeconomic determinants also have significant impact on the yield curve factors. This effect on the macroeconomic determinants is decayed with oscillations over the next 12 months.

3.6 Analysis for out-of-sample forecasting

The in-sample evidence of forecast error variance and impulse response have suggested that feedback relationships between macroeconomic determinants and yield curve factors do exist. However, the more interesting thing which investors and authorities should be concerned about is the predictability on these determinants and yield curve factors. The prediction rule is as follows.

Table 6.
Variance of decomposition of yield curve factors

Period	β_0	β_1	β_2	τ	CPIG	CBDR	IR
A. Variance of decomposition of β_0							
3	91.9849	0.8852	0.0535	6.5389	0.3885	0.1108	0.0382
6	87.2192	1.4330	1.4023	6.5309	1.7600	0.6637	0.9909
9	86.1179	1.5875	1.5781	6.7148	1.9595	0.9601	1.0822
12	85.9293	1.6081	1.5935	6.7165	2.0104	1.0220	1.1201
B. Variance of decomposition of β_1							
3	8.9725	73.8492	0.0854	12.8022	2.0049	0.9834	1.3023
6	10.2082	69.0912	0.1869	12.4944	3.4710	2.0493	2.4990
9	10.4188	67.4465	0.9131	12.6091	3.7625	2.1403	2.7098
12	10.3376	66.8850	0.9489	13.0256	3.8014	2.2239	2.7777
C. Variance of decomposition of β_2							
3	8.5326	1.1377	82.2049	2.1175	0.9225	0.6721	4.4127
6	10.3395	1.8915	75.2378	3.1446	1.4187	2.9676	5.0002
9	10.4772	2.7975	73.1891	3.3258	1.5126	2.9229	5.7749
12	10.6262	2.8228	72.3086	3.2926	1.7529	3.4334	5.7635
D. Variance of decomposition of τ							
3	2.9454	1.5408	7.3762	85.7184	1.4800	0.1516	0.7876
6	8.6063	2.6887	6.2113	77.2023	2.2477	1.2243	1.8194
9	9.8856	3.4762	5.9314	73.9645	3.1846	1.6442	1.9134
12	10.0334	3.9633	5.9245	72.9014	3.4723	1.7761	1.9290
E. Variance of decomposition of CPIG							
3	0.7819	3.5767	2.6297	0.9031	78.3904	1.7474	11.9708
6	1.6644	3.3427	4.7550	1.7275	73.2799	2.1916	13.0388
9	2.9879	3.1742	4.5929	1.7255	72.9219	2.2321	12.3655
12	2.9837	3.2073	4.9163	1.7998	72.5445	2.2655	12.2829
F. Variance of decomposition of CBDR							
3	0.5199	3.4751	0.9183	1.2312	0.6980	90.7086	2.4489
6	1.2840	6.4170	2.0440	2.1676	1.4529	82.3636	4.2709
9	1.4483	6.5330	2.1955	2.3125	1.4826	81.4046	4.6235
12	1.5542	6.6647	2.1985	2.3699	1.5124	81.1003	4.5999
G. Variance of decomposition of IR							
3	0.3294	4.8196	1.0145	0.3046	2.3617	4.2591	86.9110
6	0.8165	6.4117	1.1088	1.0818	5.8546	8.4133	76.3133
9	0.8823	6.6936	1.0965	1.7755	6.4540	8.6668	74.4314
12	1.0438	7.5360	1.1291	1.7654	6.5575	8.6001	73.3680

Note：1. The length of the period is 12 month. All variables are taken by first difference.
2. Unit in this table is percent.

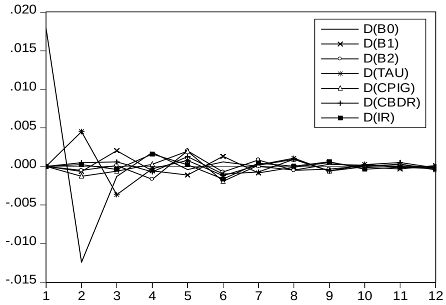

Figure 2 Month after the β_0 shock

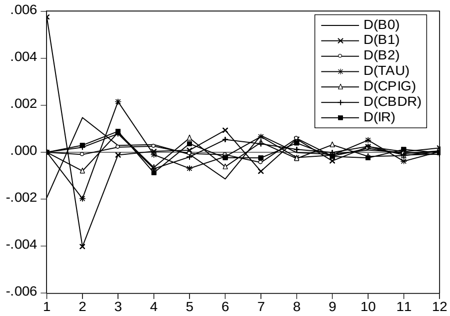

Figure 3 Month after the β_1 shock

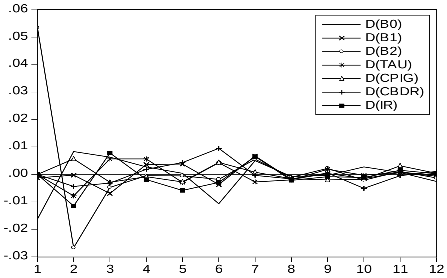

Figure 4 Month after the β_2 shock

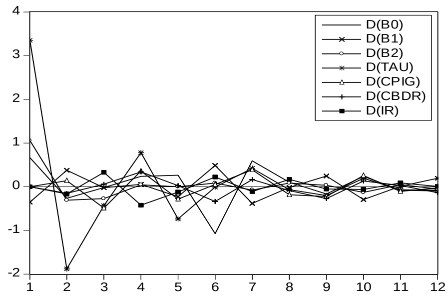

Figure 5 Month after the τ shock

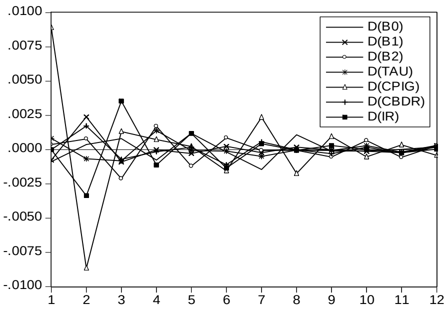

Figure 6 Month after the CPIG shock

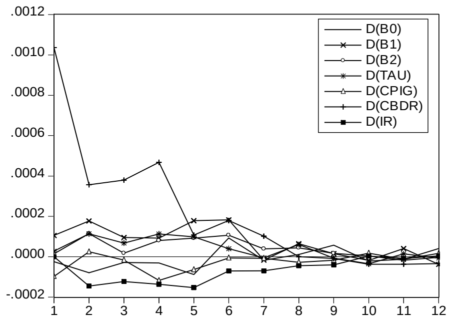

Figure 7 Month after the CBDR shock

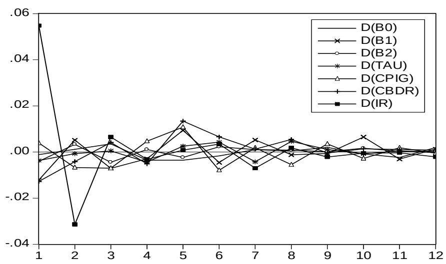

Figure 9 Month after the IR shock

Moving window technology is employed to predict the estimation on these seven variables in a row. For example, we decompose the total sample period into the rowing period and estimated period. In the rowing period, from January 2000 to December 2010, the forecasted series are generated by the VAR model based on the data of previous 48 months.

We try to examine the robustness of predictability of the VAR model with out-of-sample data. The Root Mean Squared Error (RMSE) and Hit Rate are both employed to judge the predictability of the VAR model. The comparison results are shown in Table 7. Among the yield curve factors, the predictability of β_1 has the smallest RMSE of 1.2504%, and the next smallest RMSE is shown on the predictability of β_0. Therefore, the yield curve factors, as β_1 and β_0 , are more predictable by using the VAR model. Furthermore, among the predictability of the macroeconomic determinants, the most predictable by the VAR model is the central bank discount rate, with the RMSE of 0.1025%. In addition, CPIG is the second- best predictable variable, with the RMSE of 1.8253%. The other indicator to measure the robustness of predictability of the VAR model is the Hit Rate, which presents the objective predictability by comparing the sign of the forecasted direction to each real sign of the observations. According to this indicator, we conclude that both β_0 and the central bank discount rate have the same Hit Rate of 54%, consistent with the results for RMSE.

Table 7.
Predictability of variables in out-of-sample period

Variable	β_0	β_1	β_2	τ	CPIG	CBDR	IR
RMSE	1.8079%	1.2504%	6.4374%	511.4027%	1.8253%	0.1025%	2.7156%
Hit Rate	54%	36%	41%	49%	40%	54%	34%

Note: 1. Root Mean Squared Error is defined as follow： $\sqrt{\frac{1}{n}\sum_{i=1}^{n}(Real_i\ variable - Prediction_i)^2}$ 。

2. The Hit Rate is defined at a confidence level as the probability of correct forecasting direction for the change of variables. The result of its forecasting success as a probability is shown in the second row.

To sum up the results for the out-sample-test, the VAR model incorporated in yield curve factors and macroeconomic determinants does have good predictability, especially for the variables of β_0, β_1, CBDR and CPIG. This evidence illustrates that the yield factors have richer interactions with macroeconomic determinants. Therefore, the VAR model including yield curve factors and macroeconomic determinants could offer investors and regulation authority economically important information to study future macroeconomic fundamentals and the changes of the yield curve.

4. Conclusion

This paper has solved the problem of an insufficient zero coupon bond sample in the Taiwan Bond market and employed a numerical methods, i.e. the Newton Method, to estimate the level, slope and curvature factors within the government coupon bonds based on the Nelson and Siegel (1987) model. Then, we use the yield curve factors into the VAR model with macroeconomic determinants, and examine the effect of the feedback relationship and model predictability through impulse responses, variance decomposition techniques, and the out-of-sample performance of the VAR model.

For the decomposition results of the forecast error variance, we conclude that only the short-term and mid-term yield factors have significant feedback relationships with the macroeconomic determinants, GCPI and IR. The macroeconomic determinant, CBDR, has largely been influenced by β_1 and β_2 with uni-direction. Meanwhile, the analysis of the impulse response function also has pointed out the shocks of the macroeconomic determinants and yield curve factors that also have significant impact on each other. Finally, the robust test of the predictability for the VAR model with the out-of-sample data has shown that yield factors have richer interactions with macroeconomic determinants. Thus, our findings could offer investors and the regulation authority very rich and important information to study future macroeconomic fundamentals and the changes of yield curve.

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Appendix 1
The estimated coefficients of VAR model

	d(β_0)	d(β_1)	d(β_2)	d(τ)	d(CPIG)	d(CBDR)	d(IR)
C	-0.0005	0.0005	0.0008	-0.0121	0.0000	-0.0001	0.0021
	(-0.3271)	(1.0765)	(0.1766)	(-0.0426)	(-0.0638)	(-1.1784)	(0.4628)
d β_0 (-1)	-0.7910***	0.0292	0.0888	14.7084	0.0056	-0.0012	0.0160
	(-8.2342)	(0.8920)	(0.2953)	(0.7611)	(0.1148)	(-0.2141)	(0.0512)
d β_0 (-2)	-0.6256***	0.0498	0.2300	16.3150	0.0379	-0.0028	-0.0145
	(-5.2154)	(1.2195)	(0.6127)	(0.6761)	(0.6253)	(-0.4045)	(-0.0371)
d β_0 (-3)	-0.4056***	0.0689	0.3636	19.8731	0.0261	-0.0046	-0.1325
	(-3.2557)	(1.6235)	(0.9325)	(0.7929)	(0.4134)	(-0.6296)	(-0.3274)
d β_0 (-4)	-0.2528**	0.0851**	0.4501	34.7174	0.0501	-0.0066	-0.1039
	(-2.1696)	(2.1441)	(1.2343)	(1.4810)	(0.8492)	(-0.9595)	(-0.2745)
d β_0 (-5)	-0.1213	0.0511	-0.2470	-8.1902	0.0617	0.0037	-0.3091
	(-1.2727)	(1.5746)	(-0.8283)	(-0.4272)	(1.2800)	(0.6650)	(-0.9984)
d β_1 (-1)	-0.0540	-0.7361***	-0.5488	31.0569	0.1460	0.0223	-0.2138
	(-0.2045)	(-8.1954)	(-0.6647)	(0.5851)	(1.0941)	(1.4442)	(-0.2494)
d β_1 (-2)	0.1261	-0.4636***	-1.3988	37.1415	0.1948	0.0145	-1.0695
	(0.3850)	(-4.1593)	(-1.3652)	(0.5639)	(1.1761)	(0.7545)	(-1.0057)
d β_1 (-3)	0.1951	-0.3353***	-1.0859	46.9730	0.1626	0.0025	-1.7947**
	(0.5891)	(-2.9749)	(-1.0480)	(0.7053)	(0.9708)	(0.1266)	(-1.6688)
d β_1 (-4)	0.0653	-0.1785*	-0.7902	-32.3115	0.0776	0.0142	-0.0557
	(0.2077)	(-1.6673)	(-0.8030)	(-0.5108)	(0.4881)	(0.7730)	(-0.0545)
d β_1 (-5)	0.1610	0.0096	-1.1996	48.2125	0.1070	0.0355***	-0.3618
	(0.6561)	(0.1153)	(-1.5617)	(0.9764)	(0.8621)	(2.4709)	(-0.4537)
d β_2 (-1)	-0.0357	0.0102	-0.4663***	5.1279	0.0171	0.0013	0.0374
	(-1.3014)	(1.0952)	(-5.4279)	(0.9285)	(1.2314)	(0.7979)	(0.4194)
d β_2 (-2)	-0.0264	0.0092	-0.3031***	3.4839	-0.0076	-0.0005	-0.0092
	(-0.8665)	(0.8848)	(-3.1843)	(0.5694)	(-0.4920)	(-0.2641)	(-0.0930)
d β_2 (-3)	-0.0496	0.0058	-0.2171**	-0.5179	0.0024	0.0002	0.0167
	(-1.6266)	(0.5547)	(-2.2756)	(-0.0844)	(0.1570)	(0.1234)	(0.1684)
d β_2 (-4)	-0.0148	0.0131	-0.1538*	1.3581	-0.0139	0.0007	-0.0284
	(-0.4972)	(1.3011)	(-1.6559)	(0.2274)	(-0.9270)	(0.4291)	(-0.2942)
d β_2 (-5)	-0.0197	0.0101	-0.1428*	6.3085	-0.0100	0.0014	-0.0672
	(-0.7249)	(1.0891)	(-1.6795)	(1.1541)	(-0.7284)	(0.8481)	(-0.7617)
d τ (-1)	0.0014***	-0.0006***	-0.0027**	-0.5668***	0.0000	0.0000	-0.0006
	(3.1895)	(-3.8339)	(-1.9652)	(-6.4975)	(-0.1936)	(1.0434)	(-0.4102)
d τ (-2)	0.0006	-0.0001	-0.0012	-0.4164***	-0.0002	0.0000	0.0000
	(1.2157)	(-0.7165)	(-0.6964)	(-3.8866)	(-0.7698)	(1.2004)	(-0.0033)
d τ (-3)	0.0005	0.0000	0.0006	-0.0876	-0.0001	0.0000	-0.0021
	(0.8876)	(-0.1978)	(0.3273)	(-0.7805)	(-0.1957)	(1.1632)	(-1.1406)
d τ (-4)	0.0000	-0.0001	0.0003	-0.2403**	0.0000	0.0000	-0.0016
	(0.0877)	(-0.5274)	(0.1812)	(-2.2814)	(-0.1155)	(0.5980)	(-0.9489)
d τ (-5)	-0.0001	-0.0003*	0.0007	-0.2276***	0.0001	0.0000	0.0011
	(-0.2480)	(-1.9331)	(0.4674)	(-2.5009)	(0.3260)	(0.8074)	(0.7381)
d CPIG (-1)	-0.1396	-0.0897	0.6541	14.9035	-0.9336***	0.0073	-0.6202
	(-0.8436)	(-1.5919)	(1.2630)	(0.4477)	(-11.154)	(0.7581)	(-1.1539)
d CPIG (-2)	-0.3042	-0.0456	0.4393	-32.7652	-0.7914***	0.0063	-1.6643**

	(-1.3560)	(-0.5972)	(0.6257)	(-0.7259)	(-6.9727)	(0.4812)	(-2.2837)
d CPIG (-3)	-0.3123	-0.0942	-0.0259	-8.4892	-0.5813***	-0.0080	-1.6796**
	(-1.2832)	(-1.1370)	(-0.0340)	(-0.1734)	(-4.7225)	(-0.5607)	(-2.1250)
d CPIG (-4)	-0.1096	0.0039	-0.3042	-27.1574	-0.3362***	-0.0164	-0.1587
	(-0.4754)	(0.0491)	(-0.4214)	(-0.5853)	(-2.8821)	(-1.2165)	(-0.2118)
d CPIG (-5)	-0.0710	-0.0058	0.2041	-33.7794	-0.2182***	-0.0080	-0.1033
	(-0.4491)	(-0.1074)	(0.4129)	(-1.0629)	(-2.7302)	(-0.8667)	(-0.2013)
dCBDR(-1)	0.5268	0.2662	-6.8263	-184.4468	0.9122	0.3117***	-11.0731***
	(0.3625)	(0.5382)	(-1.5013)	(-0.6310)	(1.2411)	(3.6613)	(-2.3463)
dCBDR(-2)	0.9289	1.1616**	-5.6902	54.1737	0.0741	0.2138***	3.3359
	(0.6265)	(2.3015)	(-1.2263)	(0.1816)	(0.0988)	(2.4609)	(0.6927)
dCBDR(-3)	-0.1997	-0.1894	3.6366	336.5699	-0.0869	0.2110***	-0.6906
	(-0.1342)	(-0.3738)	(0.7807)	(1.1242)	(-0.1154)	(2.4200)	(-0.1428)
dCBDR(-4)	0.6321	-0.3549	5.7333	-16.4949	-0.2677	-0.2239***	15.4012***
	(0.4244)	(-0.7001)	(1.2302)	(-0.0550)	(-0.3553)	(-2.5660)	(3.1840)
dCBDR(-5)	-0.8482	0.0212	9.2860**	-364.0153	-0.1070	0.0725	8.2334*
	(-0.5681)	(0.0416)	(1.9876)	(-1.2121)	(-0.1416)	(0.8291)	(1.6979)
dIR (-1)	0.0045	0.0054	-0.2098***	-3.0280	-0.0613***	-0.0027*	-0.5723***
	(0.1653)	(0.5918)	(-2.4804)	(-0.5569)	(-4.4809)	(-1.6759)	(-6.5193)
dIR (-2)	-0.0108	0.0188*	-0.0595	3.8556	-0.0226	-0.0022	-0.2698***
	(-0.3442)	(1.7519)	(-0.6046)	(0.6090)	(-1.4220)	(-1.2195)	(-2.6397)
dIR(-3)	0.0072	0.0187*	-0.0584	-6.8852	-0.0202	-0.0030*	-0.2170***
	(0.2387)	(1.8189)	(-0.6161)	(-1.1308)	(-1.3162)	(-1.7156)	(-2.2074)
dIR (-4)	0.0273	0.0069	-0.1947**	-7.6471	0.0089	-0.0030*	-0.1215
	(0.9620)	(0.7131)	(-2.1955)	(-1.3414)	(0.6221)	(-1.7835)	(-1.3195)
dIR (-5)	0.0228	0.0114	-0.2727***	-5.4423	0.0001	-0.0015	0.1214
	(0.8717)	(1.2781)	(-3.3285)	(-1.0333)	(0.0075)	(-0.9677)	(1.4275)

Note:

1. This table reports the results of coefficients of the VAR model. * denote rejection of the null hypothesis of different to zero at the 5%. The T-values are shown in parentheses.
2. d(β_0),d(β_1),d(β_2),d(τ),d(CPIG), d(CBDR) and d(IR) are first difference of the β_0, β_1, β_2, τ , consumer price index growth rate, central bank discount rate and inventory rate.
3. The alphabet C presents an intercept.

Appendix 2

Granger Causality test for the yield curve factors and macroeconomic variables

Granger Causality	F-Statistic	Probability	Results
$\beta_1 \rightarrow \beta_0$	1.1780	0.3104	Non-existence
$\beta_0 \rightarrow \beta_1$	0.4397	0.6450	Non-existence
$\beta_2 \rightarrow \beta_0$	0.9864	0.3750	Non-existence
$\beta_0 \rightarrow \beta_2$	0.4271	0.6530	Non-existence
$\tau \rightarrow \beta_0$	4.5514	0.0118**	Existence
$\beta_0 \rightarrow \tau$	0.3544	0.7021	Non-existence
$CPIG \rightarrow \beta_0$	0.7624	0.4681	Non-existence
$\beta_0 \rightarrow CPIG$	0.5237	0.5933	Non-existence
$\mathrm{IR} \rightarrow \beta_0$	0.3839	0.6818	Non-existence
$\beta_0 \rightarrow IR$	1.0457	0.3536	Non-existence
$CBDR \rightarrow \beta_0$	8.8260	0.0002***	existence
$\beta_0 \rightarrow CBDR$	0.4987	0.6081	Non-existence
$\beta_2 \rightarrow \beta_1$	0.0603	0.9415	Non-existence
$\beta_1 \rightarrow \beta_2$	2.5102	0.0842*	Existence
$\tau \rightarrow \beta_1$	9.1047	0.0002***	Existence
$\beta_1 \rightarrow \tau$	1.8288	0.1637	Non-existence
$CPIG \rightarrow \beta_1$	4.6941	0.0103**	existence
$\beta_1 \rightarrow CPIG$	1.0426	0.3547	Non-existence
$IR \rightarrow \beta_1$	0.0508	0.9504	Non-existence
$\beta_1 \rightarrow IR$	0.0176	0.9825	Non-existence
$CBDR \rightarrow \beta_1$	0.1101	0.8958	Non-existence
$\beta_1 \rightarrow CBDR$	1.7906	0.1699	Non-Existence
$\tau \rightarrow \beta_2$	2.0790	0.1282	Non-existence
$\beta_2 \rightarrow \tau$	0.8558	0.4267	Non-existence
$CPIG \rightarrow \beta_2$	2.0790	0.1282	Non-existence
$\beta_2 \rightarrow CPIG$	0.8558	0.4267	Non-existence
$IR \rightarrow \beta_2$	0.1201	0.8869	Non-Existence
$\beta_2 \rightarrow IR$	2.1395	0.1208	Non-existence
$CBDR \rightarrow \beta_2$	1.3971	0.2501	Non-existence
$\beta_2 \rightarrow CBDR$	0.1552	0.8564	Non-existence
$CPIG \rightarrow \tau$	0.0385	0.9622	Non-existence
$\tau \rightarrow CPIG$	0.9328	0.3954	Non-existence

$IR \rightarrow \tau$	0.0793	0.9238	Non-existence
$\tau \rightarrow IR$	0.6526	0.5220	Non-existence
$CBDR \rightarrow \tau$	0.8530	0.4279	Non-existence
$\tau \rightarrow CBDR$	0.0166	0.9835	Non-existence
$IR \rightarrow CPIG$	0.8303	0.4376	Non-Existence
$CPIG \rightarrow IR$	0.6005	0.5497	Non-Existence
$CPIG \rightarrow CBDR$	16.7887	0.0000***	Existence
$CBDR \rightarrow CPIG$	1.0717	0.3447	Non-existence
$CBDR \rightarrow IR$	0.7801	0.4600	Non-existence
$IR \rightarrow CBDR$	4.2644	0.0156**	Existence

Note:
1. This table reports the causality among the yield curve factor and macro variables for ranking the order in the VAR model. *** indicates significant at the 1%. ** indicates significant at the 5%. * indicates significant at the 10%.
2. The null hypothesis is no significant causality between two variables.
3. d(β_0),d(β_1),d(β_2),d(τ),d(CPIG), d(CBDR) and d(IR) are first difference of the β_0, β_1, β_2, τ, consumer price index growth rate, central bank discount rate and inventory rate.