



## **Revisiting the current account sustainability for the G-7 countries: The role of structural break and nonlinearity**

**Shyh-Wei Chen<sup>a</sup>**

*a.* Department of International Business, Tunghai University

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**Abstract:** We reexamine the current account sustainability for seven developed countries by taking account of different types of non-linearities in this study. For this purpose, we adopt a battery of well-known nonlinear unit root tests in the literature. Our results show that the structural break nonlinearity and size nonlinearity are critical to the current account-GDP ratios of Canada, France, Italy, Japan, the UK and the US in testing the null hypothesis of a unit root. Nevertheless, the current account-GDP ratios of the G-7 countries do not exhibit the sign nonlinearity. That is, by taking the nonlinear trend into consideration, the threshold autoregressive and momentum threshold autoregressive models do not detect any asymmetry in the response of the external debt imbalance to deviations from its long run nonlinear trend. The current account-GDP ratio of Germany is the only one that does not have any type of nonlinearity.

**Key words:** Current account; sustainability; unit root; non-linearity; structural break

**JEL:** F32, C22

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### **1. Introduction**

**E**conomists have shown considerable interest in the issue of current account sustainability, which argues that if it holds, then an economy is able to meet its intertemporal budget constraint in the long run without a drastic change in private-sector behavior or policy changes. Generally speaking, when a country runs large and persistent current account deficits for a number of years, and the deficits are financed with short-term

debt or foreign exchange reserves, and it reflects high consumption spending; concerns often arise about the sustainability of those deficits. If this country has the ability to generate sufficient trade surpluses in the future to repay existing debt, then it indicates that the country is solvent or sustainable for its current account deficits.

Trehan and Walsh(1991) show that, empirically, the stationarity of the current account is a sufficient condition for the intertemporal budget constraint to hold. After Trehan and Walsh (1991), a wealth of studies have devoted their efforts to this issue (see, for example, Apergis et al., 2000; Bergin and Sheffrin, 2000; Liu and Tanner, 2001; Arize, 2002; Baharumshah et al., 2003; Dulger and Ozdemir, 2005; Ismail and Baharumshah, 2008; Karunaratne, 2010; Nag and Mukherjee, 2012; Lau and Baharumshah, 2005; Lau et al., 2006; Kalyoncu, 2006; Holmes, 2006a, 2006b; Holmes et al., 2010; Gnimassoun and Coulibaly, 2014).<sup>1</sup> These studies either adopt linear unit root and cointegration tests or the method of a linear panel unit root or panel cointegration to test whether or not the current account imbalance is sustainable in the long run.<sup>2</sup>

Recently, researchers (see, for example, Chortareas et al., 2004; Raybaudi et al., 2004; Holmes and Panagiotidis, 2009; Kim et al., 2009; Christopoulos and León-Ledesma, 2010; Takeuchi, 2010; Chen, 2011, 2013) have turned their attention to the adoption of more sophisticated nonlinear models to test the current account's sustainability based on the studies of Leonard and Stockman (2002), Freund (2005), Taylor (2002) and Clarida et al. (2006).<sup>3</sup> Basically, the empirical evidence from this line of research indicates that, by taking the nonlinear property into account, the US (e.g., Christopoulos and León-Ledesma, 2010), Latin American countries (e.g., Chortareas et al., 2004) and Asian countries (Kim et al., 2009) are no longer in violation of current account sustainability.

Nonlinearity is a general idea and it consists of different functional forms or sources. For example, Clarida et al. (2006) consider nonlinearities in the form of threshold effects, but are the dynamics of current account adjustment dependent upon the sign of deviations from long run equilibrium? Holmes and Panagiotidis (2009) also consider nonlinearities that stem from structural breaks by using the Breitung (2002) nonparametric cointegration test.

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<sup>1</sup> Readers are referred to Chen (2011) for a brief summary of recent contributions to current account sustainability after 2000.

<sup>2</sup> The intuition for adopting the unit root test approach to test the current account sustainability is that if a country's current account deficits increase without bound for a number of years, then the probability for a country to default its external debts is high. Please refer to Section 2 for more details.

<sup>3</sup> Chortareas et al. (2004) point out that there are at least three channels that make the current account series a nonlinear process. The first source of nonlinearity is the twin-deficit channel. A second channel that leads to nonlinearity is the level of a country's indebtedness, which reflects the willingness of foreign lenders to hold domestic assets. The third channel comes from the transaction cost.

Chen (2011) employs the Markov switching model to examine whether or not the current account deficits of the G-7 countries can be characterized by a unit root process with regime switching.

To be more specific, three types of nonlinearities are widely considered in the literature. First, nonlinearity may affect the variable in the form of structural changes in the deterministic components. From an economic point of view, if the current account imbalance-to-GDP ratio is a stationary process around a nonlinear deterministic trend, then it implies a time-varying equilibrium current account-GDP ratio. From an econometric point of view, if the true data generating process is a linear process with structural breaks (e.g., Perron, 1989) or nonlinear (e.g., Pippenger and Goering, 1993; Bierens, 1997), then the traditional unit root or cointegration tests may suffer from size distortion and low power problems.

Second, sign nonlinearity could be motivated by asymmetric market friction or the action of policy-makers may also impart nonlinear adjustment dynamics. In particular, where, for example, central banks have an explicit current account imbalance-GDP ratio target (e.g., 6% of GDP), they may pay more attention to rising current account-GDP ratios than to falling ratios due to the different implications for default risk. Specifically, this implies that the current account imbalance exhibits asymmetric adjustment. It reacts in a different manner depending on the sign of the disequilibrium or shock.

Finally, Clarida et al. (2006) point out that both government policies and market forces can induce faster current account corrections when deficits reach certain ‘danger zone,’ leading to nonlinear adjustment dynamics in the current account. Christopoulos and León-Ledesma (2010) claim that changes in the agents’ perceptions regarding risk, portfolio allocation decisions, future policy changes, and transaction costs in international financial flows, etc., can lead to changes in the dynamics of current account mean reversion and hence equilibrium values of the current account. Thus, for large changes in the current account imbalance away from equilibrium we might expect the speed of mean reversion to be faster as markets (or governments) would not be willing to finance deviations from equilibrium for long periods. Thus, the process of nonlinear adjustment depends on the size of the disequilibrium.

This paper tries to contribute to the empirical literature on reexamining whether or not the deficits in the current account of six developed countries are sustainable by taking a variety of nonlinearities into consideration. In order to examine the effect of three types of

nonlinearities on current account sustainability, the empirical approach followed is threefold. First, rather than attempting to model any structural change in trend as an instantaneous trend break, we employ logistic smooth transition models proposed by Leybourne et al. (1998) and Harvey and Mills (2002) to model the nonlinearity that stems from structural breaks. These models permit the possibility of a smooth transition between two different trend paths over time.<sup>4</sup>

The second type of nonlinearity is related to the concept of asymmetric adjustment towards equilibrium, and implies that the current account reacts in a different manner depending on the sign of the disequilibrium. In order to consider the possibility of an asymmetric adjustment towards equilibrium when testing the unit root, we adopt the nonlinear unit root tests proposed by Sollis (2004) and Cook and Vougas (2009).

Finally, when the current account deficit is larger than some threshold value, market participants may view the current account as a problem and policymakers may try to reduce the size of these deficits by a sharp depreciation of the domestic currency to avoid a financial crisis.<sup>5</sup> This type of nonlinearity is well characterized by the exponential smooth transition autoregressive (ESTAR) model. Therefore, we apply the nonlinear unit root test proposed by Kapetanios et al. (2003), Rothe and Sibbertsen (2006) and Kruse (2011) to take into account the possibility of an asymmetric speed of adjustment towards equilibrium.

The major findings of this study are as follows. First, the current account series for six out of seven countries, i.e., Canada, France, Germany, Italy, Japan and the US, are not stationary processes and are thus unsustainable based on the traditional unit roots. Second, with the exception of Germany, the current account-GDP ratios of the G-7 countries exhibit structural break nonlinearity and size nonlinearity, indicating that we are inclined to accept the null hypothesis of the unit root if we overlook the structural breaks or size nonlinearity. Third, the current account-GDP ratios of the G-7 do not have sign nonlinearity, but six out of seven countries do have size nonlinearity. This implies that the policy-makers or markets care about the asymmetric speed of adjustment towards equilibrium instead of asymmetric adjustment around a threshold towards equilibrium.

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<sup>4</sup>Leybourne and Mizen (1999) point out that “when considering aggregate behavior, the time path of structural changes in economic series is likely to be better captured by a model whose deterministic component permits gradual rather than instantaneous adjustment.”

<sup>5</sup>Kim et al. (2009, p. 167) point out that “such nonlinearity implies an equilibrium level of the current account in the neighborhood of which the behavior of the current account is close to a random walk, becoming increasingly mean reverting with the absolute size of the deviation from equilibrium.”

The remainder of this paper is organized as follows. Section 2 briefly discusses the theoretical model of the current account. Section 3 introduces the econometric methodology that we employ, and Section 4 describes the data and the empirical test results. Section 5 presents the conclusions that we draw from this research.

## 2. Theoretical Background

The intertemporal model of the current account provides the optimal current account path based on the behavior of a representative agent who is infinitely-lived and smooths consumption overtime by lending or borrowing abroad. This approach considers the current account from a savings-investment perspective. Following earlier studies such as Trehan and Walsh (1991) and Hakkio and Rush (1991), let us consider an economy with the following two-period budget constraint:

$$C_t + I_t + G_t + B_t = Y_t + (1 + r_t)B_{t-1} \quad (1)$$

where  $C_t, I_t, G_t, B_t, Y_t$  and  $r_t$  are consumption, investment, government expenditure, net foreign assets, income, and the world interest rate, respectively. Rearranging (1) we have

$$B_t = (1 + r_t)B_{t-1} + Y_t - C_t - I_t - G_t = (1 + r)B_{t-1} + NX_t \quad (2)$$

where  $NX_t$  is the country's net exports defined as  $NX_t = Y_t - C_t - I_t - G_t$ . Let  $R_t = 1 + r_t$  with expected value  $E(R_{t+j} | \Omega_{t-1}) = R$  for all  $t$  and  $j \geq 1$  and  $\Omega_{t-1}$  be the information set available at time  $(t-1)$ . Following Trehan and Walsh (1991, p. 209), we may iterate this equation forward in time, solving recursively, to obtain the result that the current credit (debt) position must be offset, in expected value terms, by future deficits (surpluses). Iterating (2) forward, we can derive:

$$B_{t-1} = - \sum_{j=0}^{\infty} R^{-(j+1)} E(NX_{t+j} | \Omega_{t-1}) + \lim_{j \rightarrow \infty} R^{-(j+1)} E(B_{t+j} | \Omega_{t-1}) \quad (3)$$

We define the intertemporal national long-run budget constraint (LRBC) hypothesis so that the last term in (3) must equal zero,

$$\lim_{j \rightarrow \infty} R^{-(j+1)} E(B_{t+j} | \Omega_{t-1}) = 0 \quad (4)$$

which states that the present discounted value of the stock of assets must converge to zero as  $t$  tends to infinity. Equation (4) is also referred to as a Non-Ponzi game condition. Trehan and Walsh (1991) show that given that the current account  $CA_t = B_t - B_{t-1}$ , a sufficient

condition for equation ( 4 ) to hold is that the current account is stationary. If the growth rate of an economy is positive, then current account sustainability holds if the ratio  $y_t = \frac{CA_t}{Y_t}$  is stationary. This means that sustainability is possible with perpetual current account deficits as long as they do not grow faster than output in terms of expected value. In this case, the sustainability hypothesis implies that the debt-to-GDP ratio is constant in the long run. Non-stationarity should be interpreted as meaning that, during the sample period observed, the behavior of the current account is not compatible with the inter-temporal budget constraint (Christopoulos and León-Ledesma, 2010).

It should be noted that in Trehan and Walsh's (1991) model, the Non-Ponzi game condition (abbreviated as NPGC) is preliminarily assumed. This assumption necessarily implies that their investigation was conducted only in regard to the necessary condition and not the sufficient condition. The sample period in this paper covers almost forty years and it is a rough assumption that the sufficient condition is satisfied. This issue has already been carefully examined in Ahmed and Rogers (1995) and Matsubayashi (2005). It should also be noted that care is needed when deciding upon the probability of NPGC.

### 3. Econometric Methodology

#### 3.1 Structural Break nonlinearity with LSTR Unit Root Test

As mentioned in the previous section, nonlinearity may affect a variable in the form of structural changes in the deterministic components. That is, a broken time trend is a particular case of a nonlinear time trend. In order to take account of the possibility of nonlinear trends, we apply the Leybourne et al. (1998) (LNV hereafter) nonlinear trend modeling approach. Leybourne et al. (1998) develop a unit root test against the alternative hypothesis of stationarity around a logistic smooth transition (LSTR) nonlinear trend. It is appealing as it permits structural shifts to occur gradually over time. Leybourne et al. (1998) consider three models:

$$\text{Model A } y_t = \alpha_1 + \alpha_2 S_t(\gamma, \tau) + v_t, \quad (5)$$

$$\text{Model B } y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t(\gamma, \tau) + v_t, \quad (6)$$

$$\text{Model C } y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t(\gamma, \tau) + \beta_2 t S_t(\gamma, \tau) + v_t, \quad (7)$$

where  $v_t$  is a zero mean I(0) process,  $S_t(\gamma, \tau)$  is the logistic smooth transition function:

$$S_t(\gamma, \tau) = [1 + \exp\{-\gamma(t - \tau T)\}]^{-1} \quad (8)$$

The parameter  $\tau$  determines the timing of the transition midpoint. Since  $\gamma > 0$ , we have  $t \rightarrow -\infty, S_{-\infty}(\gamma, \tau) = 0$ ,  $t \rightarrow +\infty, S_{-\infty}(\gamma, \tau) = 1$ , and  $S_{\tau}(\gamma, \tau) = 0.5$ . The speed of transition is determined by the parameter  $\gamma$ . If  $v_t$  is a zero-mean  $I(0)$  process, then in Model A  $y_t$  is stationary around a mean which changes from the initial value  $\alpha_1$  to the final value  $\alpha_1 + \alpha_2$ . Model B is similar, with the intercept changing from  $\alpha_1$  to  $\alpha_1 + \alpha_2$ , but it allows for a fixed slope term. In Model C, in addition to the change in intercept from  $\alpha_1$  to  $\alpha_1 + \alpha_2$ , the slope also changes simultaneously, and with the same speed of transition, from  $\beta_1$  to  $\beta_1 + \beta_2$ .

The null hypothesis and alternative hypothesis are as follows:

$$H_0 \ y_t = \mu_t, \ \mu_t = \mu_{t-1} + \varepsilon_t, \ \mu_0 = \varphi \quad (9)$$

$$H_1 \ \text{Model A, Model B or Model C,} \quad (10)$$

or

$$H_0 \ y_t = \mu_t, \ \mu_t = \kappa + \mu_{t-1} + \varepsilon_t, \ \mu_0 = \varphi \quad (11)$$

$$H_1 \ \text{Model B or Model C,} \quad (12)$$

LNV suggests a two-step testing strategy, first estimating Eqs. (5)–(8) by nonlinear least squares, and then applying an ADF test with no deterministic component to the resulting residual,

$$\Delta \hat{v}_t = \hat{\rho} \hat{v}_{t-1} + \sum_{i=1}^k \hat{\delta}_i \Delta \hat{v}_{t-i} + \hat{\eta}_t \quad (13)$$

The statistics are labeled  $s_\alpha$ ,  $s_{\alpha(\beta)}$  and  $s_{\alpha\beta}$  corresponding to Models A to C, respectively.

A potential problem with the LNV test is that it allows only one-time smooth structural shift. It is highly plausible that more than one structural shift may have occurred during the observation period of the time series being investigated. Harvey and Mills (2002) adopt a double logistic smooth transition function (DLSTR) to permit two structural shifts. Their models are as follows:

$$\text{Model A} \ y_t = \alpha_1 + \alpha_2 S_{1t}(\gamma_1, \tau_1) + \alpha_3 S_{2t}(\gamma_2, \tau_2) + v_t, \quad (14)$$

$$\text{Model B} \ y_t = \alpha_1 + \beta_1 t + \alpha_2 S_{1t}(\gamma_1, \tau_1) + \alpha_3 S_{2t}(\gamma_2, \tau_2) + v_t, \quad (15)$$

$$\text{Model C } y_t = \alpha_1 + \beta_1 t + \alpha_2 S_{1t}(\gamma_1, \tau_1) + \beta_2 t S_{1t}(\gamma_1, \tau_1) + \alpha_3 S_{2t}(\gamma_2, \tau_2) + \beta_3 t S_{2t}(\gamma_2, \tau_2) + v_t, \quad (16)$$

with the logistic smooth transition function

$$S_{it}(\gamma_i, \tau_i) = [1 + \exp\{-\gamma_i(t - \tau_i T)\}]^{-1}, \quad \gamma_i > 0, \quad i=1,2 \quad (17)$$

The midpoints of the two transitions are given by  $\tau_1 T$  and  $\tau_2 T$ , respectively. The transitions' speeds are allowed to differ, and are respectively determined by  $\gamma_1$  and  $\gamma_2$ . Tests of a unit root null hypothesis against one of the above models as the alternative can be conducted using the two-step procedure employed by Leybourne et al. (1998). The statistics are called  $s_{2\alpha}$ ,  $s_{2\alpha(\beta)}$  and  $s_{2\alpha\beta}$  corresponding to Models A to C, respectively.

### 3.2 Sign nonlinearity with LSTR-TAR and LSTR-MTAR Unit Root Test

The second type of nonlinearity is related to the concept of asymmetric adjustment towards equilibrium, and implies that the current account reacts in a different manner depending on the sign of the disequilibrium. In order to consider the possibility of an asymmetric adjustment towards equilibrium when testing the unit root, we adopt the unit root test proposed by Sollis (2004) and Cook and Vougas (2009). They combine the ideas of Enders and Granger (1998) and Leybourne et al. (1998) and develop tests of the null hypothesis of a unit root, that under the alternative hypothesis allow for stationary asymmetric adjustment around a smooth transition between deterministic linear trends.

We consider two asymmetric versions in order to capture sign asymmetry. The first is the threshold autoregressive (TAR) model:

$$\Delta \hat{v}_t = I_t \hat{\rho}_1 \hat{v}_{t-1} + (1 - I_t) \hat{\rho}_2 \hat{v}_{t-1} + \sum_{i=1}^k \hat{\delta}_i \Delta \hat{v}_{t-i} + \hat{\eta}_t \quad (18)$$

where  $I_t$  is the Heaviside indicator function

$$I_t = \begin{cases} 1, & \text{if } \hat{v}_{t-1} \geq 0, \\ 0, & \text{if } \hat{v}_{t-1} < 0 \end{cases} \quad (19)$$

and  $\hat{v}_t$  is the residual from the first step by using the nonlinear least squares for equation (7).

Eqs.(5)–(7), (18) and (19) refer to the LSTR-TAR model. If  $H_0: \rho_1 = \rho_2 = 0$  in (18), then

$\hat{v}_t$  and therefore  $y_t$  contains a unit root. The statistics are referred to as  $F_\alpha$ ,  $F_{\alpha(\beta)}$  and

$F_{\alpha\beta}$  and correspond to Models A to C, respectively. Sollis (2004) shows that the F-statistic

does not have an asymptotic standard normal distribution and he tabulates the asymptotic



critical values of the  $t$  statistics via stochastic simulations. If  $H_0: \rho_1 = \rho_2 = 0$  is rejected and  $\rho_1 = \rho_2 < 0$  hold, then  $\hat{v}_t (y_t)$  is a stationary LSTR-TAR process with symmetric adjustment. If  $H_0: \rho_1 = \rho_2 = 0$  is rejected and  $\rho_1 < 0, \rho_2 < 0, \rho_1 \neq \rho_2$  holds, then  $\hat{v}_t (y_t)$  is a stationary LSTR-TAR process displaying asymmetric adjustment.

Alternatively, Cook and Vougas (2009) combine Eqs (5)–(7), (20) and (21) and propose a logistic smooth transition-momentum TAR (LSTR-MTAR) model as follows:

$$\Delta \hat{v}_t = M_t \hat{\rho}_1 v_{t-1} + (1 - M_t) \hat{\rho}_2 v_{t-1} + \sum_{i=1}^k \hat{\delta}_i \Delta \hat{v}_{t-i} + \hat{\eta}_t \quad (20)$$

where  $M_t$  is the Heaviside indicator function,

$$M_t = \begin{cases} 1, & \text{if } \Delta \hat{v}_{t-1} \geq 0, \\ 0, & \text{if } \Delta \hat{v}_{t-1} < 0 \end{cases} \quad (21)$$

The statistics are referred to as  $F_\alpha^*$ ,  $F_{\alpha(\beta)}^*$  and  $F_{\alpha\beta}^*$  corresponding to Model A to C, respectively. Critical values must be tabulated via Monte Carlo simulations.

The threshold autoregressive model (TAR) allows the degree of autoregressive decay to depend on the state of the current account imbalance, measuring the “deep” cycles. For instance, if the autoregressive decay is fast when the imbalance is above trend and slow when the imbalance is below trend, troughs will be more persistent than peaks. Likewise, if the autoregressive decay is slow when the imbalance is above trend and fast when the deficit is above trend, peaks will be more persistent than troughs. On the other hand, the momentum threshold autoregressive model (MTAR) allows the current account imbalance to display differing amounts of autoregressive decay depending on whether the imbalance is increasing or decreasing, measuring the “sharpness” of cycles (Payne and Mohammadi, 2006).

### 3.3 Size nonlinearity with ESTAR Unit Root Test

The final type of nonlinearity is related to the possibility of an asymmetric speed of adjustment towards equilibrium, i.e., the further the current account deviates from its fundamental equilibrium, the faster will be the speed of mean reversion. This implies that the current account may be a unit root process for a given threshold of values (inner regime), but a unit root when the current account reaches the outer regime. In order to take account of the possibility of an asymmetric speed of adjustment towards equilibrium when testing for the unit root, we apply the exponential smooth transition autoregressive (ESTAR) unit root tests

proposed by Kapetanios et al. (2003)(KSS hereafter), Rothe and Sibbertsen (2006) and Kruse (2011).

Kapetanios et al. (2003) have developed a new technique for the null hypothesis of a unit root against an alternative of a nonlinear but globally stationary smooth transition autoregressive process. In particular, Kapetanios et al. (2003) test for the null hypothesis of  $\gamma = 0$  in the following model:

$$\Delta y_t = \beta y_{t-1} [1 - \exp(-\gamma y_{t-1}^2)] + \varepsilon_t \quad (22)$$

The test is carried out by a t-test of the coefficient of  $y_{t-1}^3$  being zero in the auxiliary regression

$$\Delta y_t = \delta y_{t-1}^3 + \sum_{j=1}^p \rho_j \Delta y_{t-j} + \eta_t \quad (23)$$

with the  $p$  augmentations in order to correct for serially correlated errors. The null hypothesis to be tested with Eq. (23) is  $H_0: \delta = 0$  (unit root in outer regime) against the alternative of  $H_1: \delta < 0$  (stationarity in outer regime). Kapetanios et al. (2003) show that the t-statistic for  $\delta = 0$  against  $\delta < 0$  does not have an asymptotic standard normal distribution and they tabulate the asymptotic critical values of the t statistics via stochastic simulations. In the presence of constants and trends, the data are first demeaned or detrended. We refer to this test as the KSS nonlinear augmented Dickey-Fuller test and label it as  $KSS(t_{NL})$ .

Rothe and Sibbertsen (2006) propose a Phillips-Perron-type, semi-parametric testing procedure to distinguish a unit root process from a mean-reverting exponential smooth transition autoregressive one. The test statistic is as follows:

$$Z_{NL}(t) = \frac{\hat{\sigma}}{\hat{\lambda}} t_{\hat{\beta}} - \frac{3}{2} \sum_{t=1}^T y_{t-1}^2 (\hat{\lambda}^2 - \hat{\sigma}^2) (\hat{\lambda}^2 \sum_{t=1}^T y_{t-1}^6)^{-1/2} \quad (24)$$

where  $\hat{\lambda}^2$  is the consistent estimator of the long run variance  $\lambda^2$ ,  $\hat{\sigma}^2$  is the consistent estimator of the variance  $\sigma^2$ . Their simulation results show that the power of  $Z_{NL}(t)$  dominates that of  $KSS(t_{NL})$  in the case of where  $\gamma$  is small and where the error sequence is an MA(1).

Kruse (2011) proposes an extension of the KSS unit root test, which relaxes the assumption of a zero location parameter  $c$ , i.e., Kruse (2011) considers the following modified ADF regression

$$\Delta y_t = \beta y_{t-1} \left[ 1 - \exp(-\gamma y_{t-1}^2 - c) \right] + \varepsilon_t \quad (25)$$

Following KSS, it is possible to obtain a Taylor approximation of Eq. (25) as follows:

$$\Delta y_t = \delta_1 y_{t-1}^3 + \delta_2 y_{t-1}^2 + \sum_{j=1}^p \rho_j \Delta y_{t-j} + \mu_t \quad (26)$$

Eq. (26) incorporates lags of the dependent variable in order to eliminate serial correlation in the error terms. In order to test the null hypothesis of a unit root,  $H_0: \delta_1 = \delta_2 = 0$  against a globally stationary ESTAR process,  $H_1: \delta_1 < 0, \delta_2 = 0$ , Kruse (2011) proposes a  $\tau$  test, which is a version of the Abadir and Distaso (2007) Wald test.

#### 4. Data and Results

The data include quarterly observations of the current account imbalance as percentages of GDP. We consider the G-7 industrial countries, i.e., Canada, France, Germany, Italy, Japan, the UK and the US in our empirical study. The sample periods are different across countries depending on the availability of data. The sample period is 1970:Q1–2012:Q1 for Canada, Japan, the UK and the US; 1971:Q1–2012:Q1 for Italy and Germany and 1975:Q1–2012:Q1 for France. All data are obtained from Datastream.

As a preliminary analysis, we apply a battery of unit root tests to determine the order of integration of the current account imbalance-GDP ratio. We consider the Augmented Dickey-Fuller (ADF) test, as well as the ADF-GLS test of Elliott et al. (1996) in this study. Vougas (2007) highlights the usefulness of the Schmidt and Phillips (1992) (SP hereafter) unit root test in practice. Therefore, we also employ it in this study. These authors propose some modifications of existing unit root tests in order to improve their power and size. An auxiliary regression is run with an intercept and a time trend. To select the lag length ( $k$ ) we use the ‘t-sig’ approach proposed by Hall (1994). That is, the number of lags is chosen for which the last included lag has a marginal significance level that is less than the 10% level.

The results of applying these tests are reported in the top panel of Table 1. We find that, with the exception of the UK, the null hypothesis of a unit root cannot be rejected at the 5% level for any of the unit root statistics. In addition, the SP test (see Schmidt and Phillips, 1992), with parametric correction, cannot reject the unit root hypothesis with both a linear and quadratic trend at the five percent significance level, suggesting that the current account-GDP ratios for six out of the G-7 countries are non-stationary processes. Based on

the linear unit root test results, only the UK shows evidence of the current account-GDP ratio being a stationary process and therefore being sustainable.

As Perron (1989) pointed out, in the presence of a structural break, the power to reject a unit root decreases if the stationary alternative is true and the structural break is ignored. To address this issue, we use Zivot and Andrews' (1992) sequential one trend break model and Lumsdaine and Papell's (1997) two-trend breaks model to investigate the order of the empirical variables. We use the 't-sig' approach proposed by Hall (1994) to select the lag length ( $k$ ). We set  $k_{\max} = 12$  and use the approximate 10% asymptotic critical value of 1.60 to determine the significance of the t-statistic on the last lag. We use the 'trimming region'  $[0.10T, 0.90T]$  and select the break point endogenously by choosing the value of the break that maximizes the ADF t-statistic. We report the results in the bottom panel of Table 1. The results suggest that, for all countries, the null hypothesis of a unit root cannot be rejected at the 5% significance level, indicating that the current account-GDP ratios are non-stationary in their respective levels. These findings fully echo those obtained from the linear unit roots.

We can categorize the hypotheses into four cases:  $H_1$ : linear and stationary process.  $H_2$ : linear and unit-root non-stationary process.  $H_3$ : nonlinear and stationary process.  $H_4$ : nonlinear and unit-root non-stationary process. It is embarrassing that when applying the SP and ADF-GLS tests for  $H_2$  against  $H_1$ , two (Italy and the UK) out of seven countries reject the existence of unit root and the current-account-GDP ratios of these countries can be considered as stationary. Then comes the question: should we test for  $H_1$  against  $H_3$  rather than  $H_2$  against  $H_3$ ? In order to validate then on linear unit root used in this paper, we conduct several nonlinearity tests (i.e.,  $H_1$  vs  $H_3$ ) for the current account-GDP ratio. Psaradakis and Spagnolo (2002) examines the relative performance of some popular nonlinearity tests. The nonlinearity tests considered include the RESET-type tests, the Keenan test, the Tsay test, the McLeod-Li test, the BDS test, the White dynamic information matrix test, and the neural network test.<sup>6</sup> We adopt these statistics to examine whether there any nonlinearity exists in the current account-GDP ratio. The results are reported in Table 2. Table 2 shows that, except for the Germany, the UK and the US, some of the p-values of these nonlinear tests are smaller than the 10% significance level or better, indicating that the current account-GDP ratios of the these countries have nonlinear

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<sup>6</sup> Readers are referred to Psaradakis and Spagnolo (2002) for detailed descriptions of these tests.

property.<sup>7</sup>

As mentioned in the Introduction, nonlinearity occurs in the form of structural changes in the deterministic components. That is, a broken time trend is a particular case of a nonlinear time trend. In order to take the possibility of nonlinear trends into consideration, we apply the logistic smooth transition unit root, championed by Leybourne et al. (1998), in this study. This approach permits structural shifts to occur gradually over time instead of instantaneously. We summarize the test results in the top panel of Table 3. Based on the  $s_{\alpha}$ ,  $s_{\alpha(\beta)}$  and  $s_{\alpha\beta}$  statistics, it is found that for France, Italy, Japan and the UK, the null hypothesis of a unit root is rejected at least at the 10% significance level or better, indicating that the current account-GDP ratios of the four countries are stationary processes around a logistic smooth transition nonlinear trend.

Figure 1 includes the time series plots of current account-GDP ratios (black line) and the estimated logistic smooth transition functions (blue line) of Model C for the G-7.<sup>8</sup> Intuitively, if the true data generating process follows the logistic smooth transition nonlinear process, then the estimated logistic smooth transition trends are close to the raw data. As such, it is highly possible to reject the null hypothesis of non-stationarity. Taking France as an example, the estimated logistic smooth transition trend of Model C is quite close to the raw data. These plots echo the rejections of the null hypothesis of a unit root by the  $s_{\alpha(\beta)}$  and  $s_{\alpha\beta}$  statistics as shown in the top panel of Table 3. This is true for the cases of Italy, Japan and the UK. However, this is not true for the cases of Canada, Germany and the US. A possible reason is that a one-time smooth structural shift is not good enough to capture the dynamic processes of the current account-GDP ratios of these countries. We therefore turn to estimate the double logistic smooth transition model, Eqs (14)–(17), and we summarize the test results in the bottom panel of Table 3.

As shown in Table 3, based on the  $s_{2\alpha}$  statistics, the null hypothesis of a unit root is rejected at the 1% significance level for Italy. According to the  $s_{2\alpha(\beta)}$  and  $s_{2\alpha\beta}$  statistics, the unit root is rejected at the 10% significance level or better for Canada and France. The current account-GDP ratio of the US also follows the nonlinear smooth transition stationary process since the  $s_{2\alpha\beta}$  statistic is rejected at the 10% level. The corresponding estimated

<sup>7</sup> It should be noted that linearity or nonlinearity has no bearing on current account sustainability as long as it is stationary.

<sup>8</sup> The plots of Models A and B for the G-7 and detailed estimation results of the logistic smooth transition models, i.e., Eqs (5)–(8), are available from the author upon request.

logistic smooth transition functions for the G-7 are included in Figure 2.<sup>9</sup> If we observe these graphs with unaided eye and compare them to Figures 1, we find that the estimated logistic smooth transition functions based on the DLSTR model can accurately identify the turning points of the raw data, especially for the cases of Canada, France, Germany, Italy and the US. In other words, the “identification performance” for the structural shifts or turning points of the DLSTR model (with two smooth structural shifts) is better than that of the LSTR model (with a one-time smooth structural shift).

Let us turn our attention to the sign nonlinearity of the current account-GDP imbalance. The LSTR-TAR specification is examined by first testing the null hypothesis of a unit root,  $H_0 : \rho_1 = \rho_2 = 0$  in equation (18) and comparing the appropriate critical values from Sollis (2004). The results are included in the top panel of Table 4. From the results for  $F_\alpha^*$ ,  $F_{\alpha(\beta)}^*$  and  $F_{\alpha\beta}^*$  as shown in Table 4, for France, Italy and Japan, the null hypothesis of a unit root is rejected at the 10 percent significance level or better. The results imply that the current account-GDP ratios are nonlinear stationary processes for these countries. However, the null hypothesis of symmetry,  $\rho_1 = \rho_2$ , is not rejected at the conventional significance level. Thus, the evidence suggests that the ‘deep’ cycles (adjustments) around the threshold value of the current account-GDP imbalances are symmetric.

Next, the MTAR specification, which has favorable power and size properties relative to the alternative of symmetric adjustment (Enders and Siklos, 2001, p. 166), is examined. The LSTR-MTAR model allows the adjustment to depend on the previous period’s change in the current account deficit. The results of the test statistics ( $F_\alpha^*$ ,  $F_{\alpha(\beta)}^*$  and  $F_{\alpha\beta}^*$ ) for the LSTR-MTAR model are reported in the bottom panel of Table 4. For France, Italy and Japan, the null hypothesis of a unit root,  $H_0 : \rho_1 = \rho_2 = 0$ , in equation (20) is rejected at the 10 percent or better significance level. These findings indicate that the current account-GDP ratios for these countries are characterized, again, by nonlinear stationarity. The null hypothesis of symmetry,  $\rho_1 = \rho_2$ , is, again, not rejected at the 10 percent significance level or better. Thus, it appears that the ‘sharpness’ cycles (adjustments) around the threshold value of the current account-GDP ratios of these countries are symmetric.

In sum, the empirical results for the respective LSTR-TAR and LSTR-MTAR models

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<sup>9</sup> The detailed estimation results of the double logistic smooth transition model, i.e., Eqs (14)–(17), are available from the author upon request.

reveal that the current account-GDP ratios for France, Germany, Italy and Japan are sustainable after taking account of the nonlinear trend. Nevertheless, they adjust symmetrically around the threshold value after taking account of the nonlinear trend.

Finally, we examine the size nonlinearity which is related to the possibility of an asymmetric speed of adjustment towards equilibrium. That is, the further the current account deviates from its fundamental equilibrium, the faster will be the speed of mean reversion. We apply the  $KSS(t_{NL})$  (Kapetanios et al., 2003),  $Z_{NL}(t)$  (Rothe and Sibbertsen, 2006) and  $\tau$  (Kruse, 2011) statistics to the raw data, and demeaned and detrended data for the current account-GDP ratio. The results are reported in Table 5, and point to the rejection of the null hypothesis of unit root against the alternative of a globally stationary ESTAR process around a nonlinear deterministic trend in 6 of the 7 countries. This implies that the size nonlinearity is a vital feature of the current account-GDP ratios of Canada, France, Italy, Japan, the UK and the US. If we overlook this feature, then we will be inclined to reach a spurious conclusion that the current account imbalance is a non-stationary process and is thus not sustainable. In fact, it is a nonlinear mean-reverting process that favors the sustainability hypothesis.

Compared to previous researches, a unique contribution of our paper is that we consider the three types of nonlinearities of the current-account deficit simultaneously. For example, Chen (2010a) examines whether or not the current account deficits for the G-7 can be characterized by a unit root process with regime switching. The evidence from the Markov switching unit root regression suggests that it is very likely that the likelihood of the LRBC holding is high for Germany and Japan, and thus the current account deficits are most likely to be sustainable. It is, however, very likely that the LRBC will not hold for Canada, France, Italy, the UK or the US, thus signifying a red signal that the current account deficits observed during the period were probably not on a sustainable path.

## 5. Concluding Remarks

This paper examines three types of nonlinearities, i.e., nonlinearity stemming from structural breaks, sign nonlinearity and size nonlinearity, of the current account imbalances for the G-7 nations and tries to answer the question: Which one is essential to enabling the current account-GDP ratio to be sustainable? For the readers' information, we summarize all of the empirical evidence of this paper in Table 6 and reach the following key conclusions.

First, by using a battery of univariate unit root tests, we find evidence in favor of non-

stationary current account-GDP ratios in six (Canada, France, Germany, Italy, Japan and the US) of the G-7 countries. The results are consistent with the previous literature (e.g., Chortareas et al., 2004; Kimet al., 2009; Chen, 2011a) in that the traditional linear unit root is inclined to accept the null hypothesis of non-stationarity, indicating that the long run intertemporal budget constraint of the current account imbalance does not hold.

Second, the current account-GDP ratios of Canada, France, Italy, Japan, the UK and the US exhibit structural break nonlinearity and size nonlinearity, indicating that both are essential in testing the null hypothesis of a unit root. If we overlook structural break nonlinearity or size nonlinearity in testing, then we will be inclined to accept the null of non-stationarity and wrongly conclude that the sustainability hypothesis does not hold.

Third, the current account-GDP ratios for the G-7 countries do not exhibit sign nonlinearity, but six out of seven countries exhibit size nonlinearity. This implies that the policy-makers or markets care about the asymmetric speed of adjustment towards equilibrium, instead of the asymmetric adjustment around a threshold towards equilibrium.

Finally, for Germany, even though none of the nonlinearity is vital to enabling the current account imbalance to be sustainable, the estimated nonlinear trend based on the double logistic smooth transition function does replicate the raw data very well. Of course we can extend the Harvey and Mills (2002) double logistic smooth transition function to the triple logistic smooth transition function, that is, we permit three structural shifts in the data. We then estimate this model and present the fitted graph in Figure 3. It is found that the estimated nonlinear trend based on the triple logistic smooth transition function perfectly overlays the raw data (see Model C). However, in order to test the null hypothesis of non-stationarity we need to develop a new test statistic and simulate the critical values accordingly. This is beyond the scope of the present work. We leave this as a research avenue in the future.

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**Table 1: Results of the linear unit root tests**

Country	Linear trend		
	ADF	SP(1)	DF-GLS
Canada	-2.115	-2.142	-2.429
France	-1.011	-1.806	-1.484
Germany	-2.052	-2.099	-2.084
Italy	-2.432	-2.992	-3.138*
Japan	-3.363*	-2.989	-2.585
UK	-3.245*	-4.325**	-3.3510**
US	-2.489	-2.018	-1.935

Country	Quadratic trend and breaks tests		
	SP(2)	ZA, Model C	LP, Model C
Canada	-2.838	-3.546	-4.312
France	-2.874	-4.173	-5.884
Germany	-2.312	-3.947	-4.375
Italy	-3.355	-4.708	-6.182
Japan	-3.379	-4.601	-5.338
UK	-3.608*	-4.089	-5.345
US	-2.233	-3.481	-4.757

(1) \*, \*\*, \*\*\* denote significance at the 10%, 5% and 1% levels, respectively. (2) ADF, SP(1) and DF-GLS denote the augmented Dickey-Fuller test, Schmidt-Phillips  $\tau$  test with linear trend and Elliott et al. (1996) DF-GLS test, respectively. (3) SP(2), ZA and LP denote the Schmidt-Phillips  $\tau$  test with quadratic trend, Zivot and Andrews (1992) and Lumsdaine and Papell (1997) tests, respectively. (4) The 5% critical values for the ADF, SP(1) and DF-GLS tests are  $-3.43$ ,  $-3.04$  and  $-2.89$ , respectively. (5) The 5% critical values for the SP(2), ZA and LP tests are  $-3.55$ ,  $-5.08$  and  $-6.75$ , respectively.

**Table 2: p-values for a battery of nonlinear tests**

	Canada	France	Germany	Italy	Japan	UK	US
RESET1	0.069	0.020	0.260	0.276	0.052	0.628	0.356
RESET2	0.069	0.632	0.260	0.276	0.031	0.209	0.356
KEENAN	0.336	0.849	0.747	0.485	0.752	0.290	0.900
TSAY	0.334	0.401	0.746	0.484	0.570	0.251	0.900
MCLEOD	0.379	0.003	0.554	0.096	0.334	0.163	0.999
BDS	0.463	0.000	0.243	0.105	0.392	0.563	0.296
WHITE1	0.366	0.130	0.236	0.055	0.612	0.527	0.764
WHITE2	0.255	0.016	0.253	0.056	0.016	0.320	0.821
NEURAL1	0.071	0.520	0.215	0.104	0.083	0.107	0.148
NEURAL2	0.055	0.195	0.851	0.105	0.245	0.128	0.162

(1) RESET1: Ramsey and Schmidt (1976). (2) RESET 2: Thursby and Schmidt (1977). (3) KEENAN: Keenan (1985). TSAY: Tsay (1986). (4) MCLEOD: McLeod and Li (1983). (5) BDS: Brock et al. (1996). (6) WHITE1 and WHITE2 are White's (1987) information matrix tests. (7) NEURAL1 and NEURAL2 are the neural network proposed by White (1989a,b).



**Table 3: Results of the LSTR and DLSTR unit root tests**

LSTR			
Country	$S_{\alpha}$	$S_{\alpha(\beta)}$	$S_{\alpha\beta}$
Canada	-2.885	-3.246	-3.201
France	-3.109	-4.709**	-5.915**
Germany	-2.434	-3.210	-3.163
Italy	-3.884*	-3.687	-5.016*
Japan	-4.681**	-4.738**	-4.785*
UK	-3.880*	-3.320	-3.507
US	-2.843	-3.076	-3.006
DLSTR			
Country	$S_{2\alpha}$	$S_{2\alpha(\beta)}$	$S_{2\alpha\beta}$
Canada	-3.513	-6.125**	-6.290**
France	-2.570	-5.621*	-5.945*
Germany	-3.745	-3.763	-5.062
Italy	-5.908**	-4.571	-5.356
Japan	-4.702	-5.138	-4.830
UK	-4.275	-4.236	-4.372
US	-2.997	-4.927	-5.944**

(1) \*, \*\*, \*\*\* denote significance at the 10%, 5% and 1% levels, respectively. (2) LSTR denotes the nonlinear unit root test proposed by Leybourne et al. (1998). (3) DLSTR denotes the nonlinear unit root test proposed by Harvey and Mills (2002). (4) The critical values for the LSTR and DLSTR statistics are obtained from Leybourne et al. (1998) and Harvey and Mills (2002).

**Table 4: Results of the LSTR-TAR and LSTR-MTAR unit root tests**

LSTR-TAR			
Country	$F_{\alpha}(\rho_1 = \rho_2)$	$F_{\alpha(\beta)}(\rho_1 = \rho_2)$	$F_{\alpha\beta}(\rho_1 = \rho_2)$
Canada	4.246(0.646)	3.646(0.719)	3.756(0.737)
France	4.800(0.959)	11.251*(0.516)	17.373**(0.939)
Germany	1.573(0.942)	5.194(0.266)	4.466(0.571)
Italy	7.735(0.898)	6.753(0.997)	12.599*(0.782)
Japan	10.988**(0.673)	11.254*(0.669)	11.781*(0.401)
UK	7.694(0.249)	5.926(0.318)	7.101(0.176)
US	4.351(0.424)	4.843(0.603)	4.522(0.795)
LSTR-MTAR			
Country	$F_{\alpha}^*(\rho_1 = \rho_2)$	$F_{\alpha(\beta)}^*(\rho_1 = \rho_2)$	$F_{\alpha\beta}^*(\rho_1 = \rho_2)$
Canada	5.457(0.114)	5.484(0.057)	5.583(0.059)
France	6.049(0.128)	11.079*(0.723)	17.461**(0.702)
Germany	1.672(0.702)	5.437(0.202)	11.454(0.000)
Italy	8.165*(0.372)	7.743(0.178)	12.899*(0.442)
Japan	10.903*(0.865)	11.195*(0.779)	11.380(0.931)
UK	7.111(0.604)	5.488(0.870)	6.118(0.912)
US	4.169(0.586)	4.783(0.691)	4.660(0.567)

(1) \*, \*\*, \*\*\* denote significance at the 10%, 5% and 1% levels, respectively. (2) LSTR-TAR denotes the nonlinear unit root tests proposed by Sollis (2004). (3) LSTR-MTAR denotes the nonlinear unit root tests proposed by Cook and Vougas (2009). (4) The numbers in the parentheses are the p-values for testing the null hypothesis of  $\rho_1 = \rho_2$ . (5) The critical values for the LSTR-TAR and LSTR-MTAR statistics are obtained from Sollis (2004) and Cook and Vougas (2009).

**Table 5: Results of the ESTAR-type unit root tests**

Country	$KSS(t_{NL})$		
	raw data	demean	Detrend
Canada	-2.503**	-2.870*	-4.038**
France	-2.669**	-3.221**	-2.741
Germany	-1.222	-1.923	-2.681
Italy	-3.199**	-3.271*	-2.890
Japan	-1.925*	-3.108**	-4.132**
UK	-1.742	-2.316	-1.957
US	-0.966	-1.990	-3.531**

Country	$Z_{NL}(t)$		
	raw data	demean	Detrend
Canada	-2.429**	-2.935**	-3.829**
France	-2.268**	-2.315	-2.294
Germany	-1.225	-2.058	-3.044
Italy	-3.139**	-3.410**	-2.908
Japan	-1.524	-2.321	-2.933
UK	-2.010*	-3.224**	-2.626
US	-1.030	-2.088	-3.280*

Country	$\tau$		
	raw data	demean	Detrend
Canada	11.005**	11.151**	16.238**
France	8.092*	11.488**	8.561
Germany	3.586	6.242	7.478
Italy	11.144**	11.177**	8.706
Japan	8.704*	12.224**	16.979**
UK	3.989	5.408	5.100
US	2.212	6.228	13.630**

(1) \*, \*\*, \*\*\* denote significance at the 10%, 5% and 1% levels, respectively.

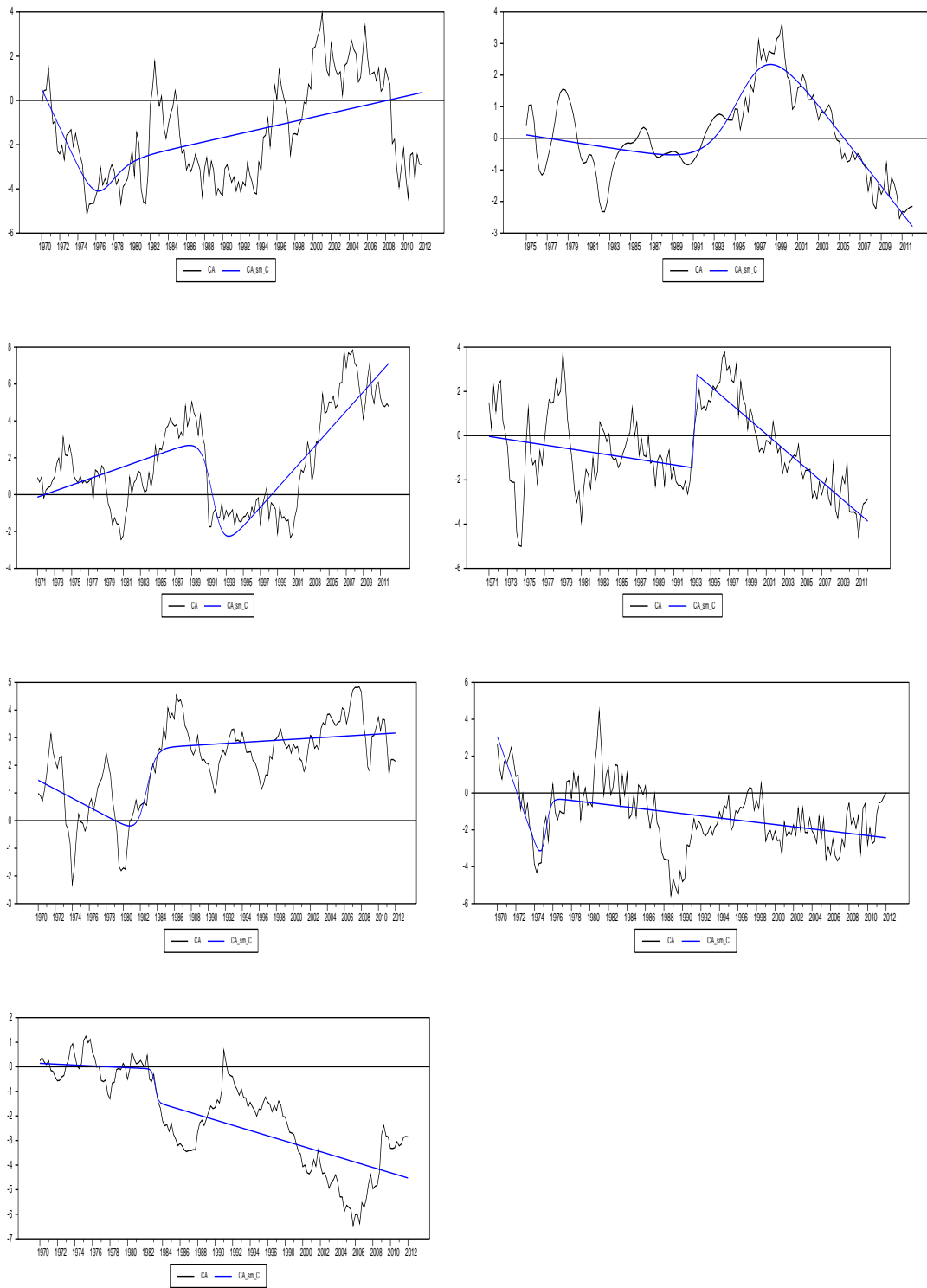
(2)  $KSS(t_{NL})$ : Kapetanios et al. (2003).  $Z_{NL}(t)$ : Rothe and Sibbertsen (2006).  $\tau$ : Kruse (2011). (3) The critical values for the three statistics are obtained from Kapetanios et al. (2003) and Kruse (2011).

**Table 6: Summary of a variety of nonlinearities**

Country	linear	SB nonlinearity		Sign nonlinearity		Size nonlinearity		
		LSTR	DLSTR	LSTR-TAR	LSTR-MTAR	$KSS(t_{NL})$	$Z_{NL}(t)$	$\tau$
Canada			yes			yes	Yes	Yes
France		yes	yes	Yes/no	Yes/no	yes	Yes	Yes
Germany								
Italy		yes	yes	Yes/no	Yes/no	yes	Yes	Yes
Japan		yes		Yes/no	Yes/no	yes		
UK	yes	yes					Yes	
US			yes			yes	Yes	Yes

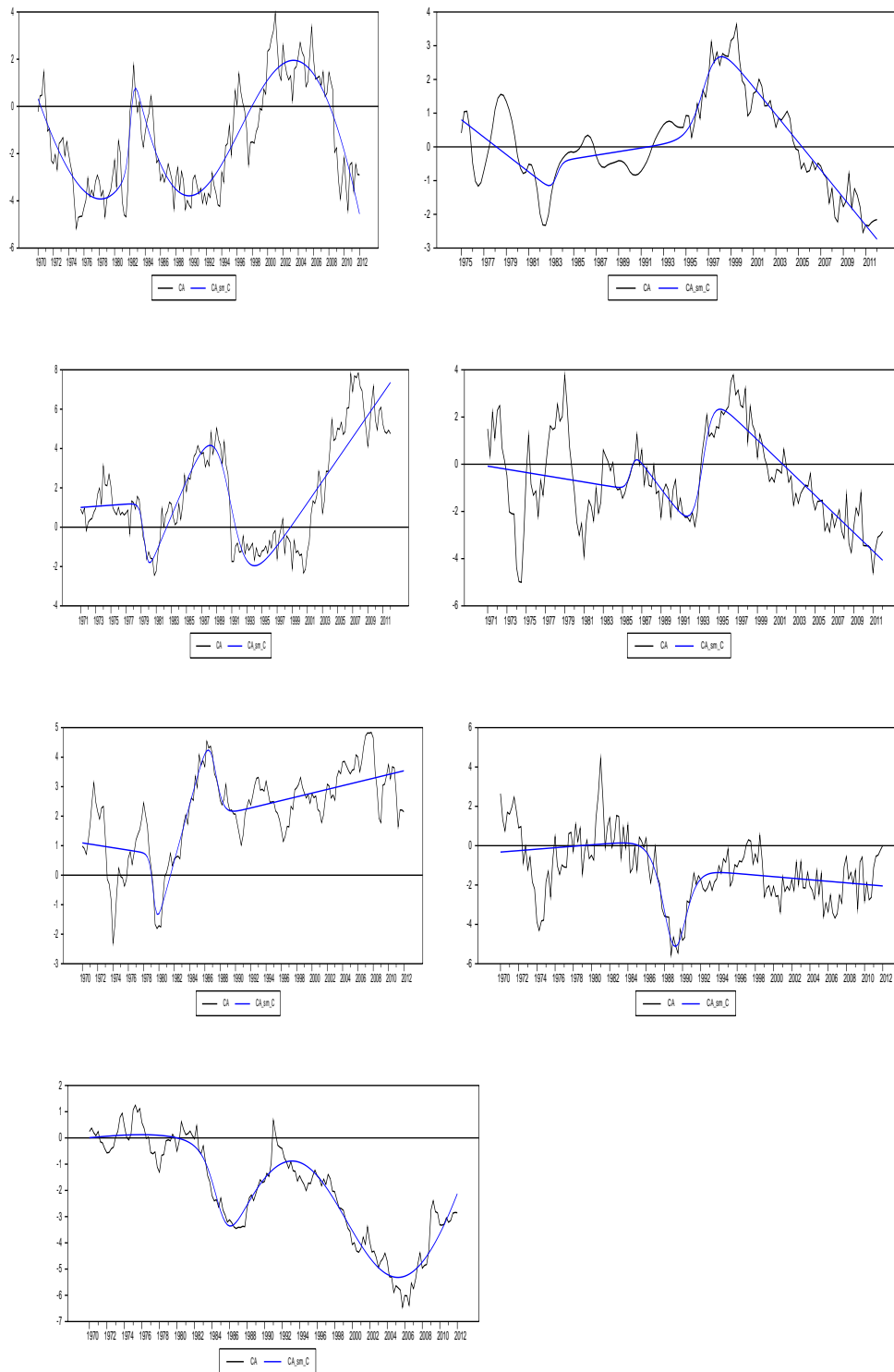
(1) SB nonlinearity indicates that nonlinearity stems from structural breaks. (2) The term “yes” indicates that the null of a unit root is rejected and in favor of linear or nonlinear stationary process. (3) The term “yes/no” indicates that we reject the null of non-stationarity favoring the sustainability hypothesis but the current account-GDP ratios adjust symmetrically around the threshold value after taking account of the nonlinear trend.

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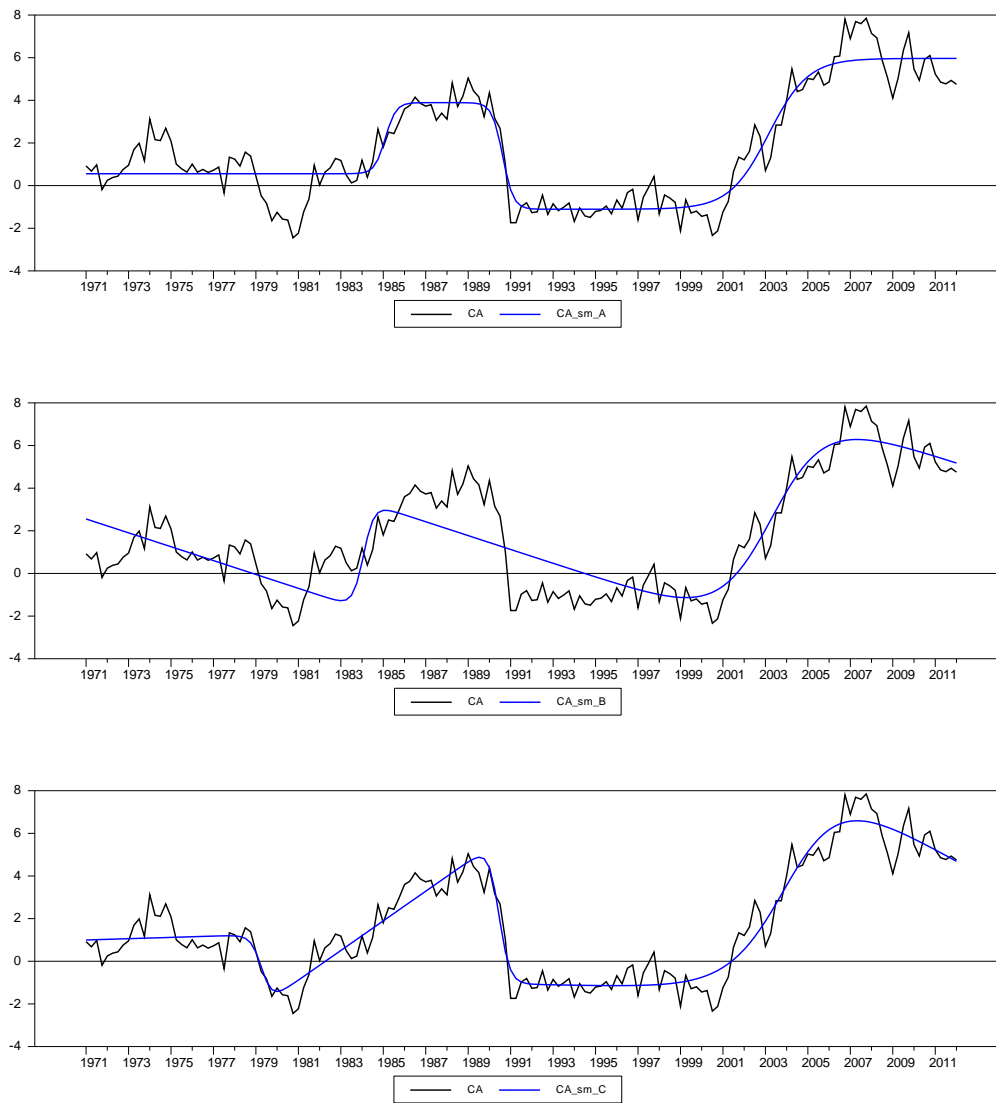


**Figure 1: Current account-GDP ratio (black line) and the fitted logistic smooth transition function(blue line) For Models C. The order of countries left to right are Canada, France, Germany, Italy, Japan, UK,US.**

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**Figure 2: Current account-GDP ratio (black line) and the fitted logistic smooth transition function (blue line) for Model C. The order of countries from left to right are Canada, France, Germany, Italy, Japan, the UK and the US.**



**Figure 3: Current account-GDP ratio (black line) and the fitted triple logistic smooth transition function (blue line) for Models A to C: Germany.**