



## **Nonlinear mean reversion in the consumption-income ratio: New evidence from the OECD countries**

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**Abstract:** Whether or not the average propensity to consume (APC) is a non-stationary process has important theoretical implications. If the APC exhibits mean reversion, then this implies that an equilibrium relationship exists between consumption and income. In this article we examine whether or not the consumption-income ratios of twenty OECD countries can be characterized by a stationary process with a non-linear trend and asymmetric adjustment. For this purpose, we employ Chen and Xie's (2015) new tests. Among the main results, it is found that the consumption-income ratios of these OECD countries are stationary after taking into account asymmetry and the non-linear trend in the long run. Our results are in line with the validity of some consumption theories such as the relative income hypothesis, the permanent income hypothesis, and the life cycle hypothesis.

**Keywords:** consumption-income ratio, stationarity, asymmetry, structural break

**JEL classification:** E21, C22

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## 1. Introduction

The stochastic property of the consumption-income ratio or average propensity to consume (APC) has long been a focus of research and policy debate in macroeconomics. In principle, whether the consumption-income ratio exhibits mean reversion or not will affect the empirical modeling of consumption functions, our understanding of savings behavior and business cycles, and economic policy. From the theoretical background, there are two opponent hypotheses about the stochastic properties of the consumption-income ratio. As noted by Sarantis and Stewart (1999), the Keynesian absolute income hypothesis and Deaton's (1977) involuntary savings theory imply a non-stationary consumption-income ratio. This would support the lack of mean reversion in the APC towards a steady-state level in response to shocks. By contrast, the relative income hypothesis (Duesenberry, 1949), the permanent income hypothesis (Friedman, 1957), the life cycle hypothesis (Ando and Modigliani, 1963) and the habit persistent model (Gale, 1973) imply a constant APC in the long run. As a result, they predict the existence of a long-run equilibrium relationship between consumption and income, thus leading the consumption-income ratio to converge to a steady-state level. In addition, growth models rendering balanced growth outcomes, which form the basis of the great-ratios literature (e.g., Blanchard and Quah, 1989; King et al., 1991) predict the existence of stationary investment-income and consumption-income ratios along the balanced growth path in the long run.<sup>1</sup>

A myriad of studies have devoted many efforts to this issue. Early studies employ the conventional unit root tests such as the ADF statistic to investigate the mean-reverting behavior of the consumption-income ratio (Ungern-Sternberg, 1986; Drobny and Hall, 1989; King et al., 1991; Molana, 1991; Hall and Patterson, 1992; Horioka, 1997; Campbell, 1987; Slesnick, 1998; Lettau and Ludvigson, 2001). As Perron (1989) pointed out, in the presence of a structural break, the power to reject a unit root decreases if the stationary alternative is true and the structural break is ignored. Hence, soon the unit root test with structural breaks become a predominant (e.g., Cook, 2005; Gomes, 2009; Romero-Avila, 2009). Motivated by the statistical power of the advances in panel unit root tests (Maddala and Wu, 1999; Hardi, 2000; Smith et al., 2004), an increasing number of authors have applied these new tools to test whether or not the consumption-income ratio is a mean-reverting process in the long run, for example, Sarantis and Stewart (1999), Romero-Avila (2008, 2009), Liao et al. (2010) and Cerrato et al. (2013), to name a few.

An important feature of previous studies is that distinct results based on previous research are due to differences in methodology, approaches and samples and are subject to diverse interpretations, thus making it difficult to reach a corroborative position on the stationarity property of the consumption-income ratio. Much evidence shows that the consumption-income ratio is non-stationary.<sup>2</sup> As noted by Cerrato et al. (2013), one explanation is that, if the consumption-income ratio declines at a decreasing rate through time (implied by the absolute income hypothesis), the APC will likely feature a linear trend. Another potential explanation for the non-stationarity of the consumption-income ratio is that the assumptions required for a constant APC do not hold.

The second potential problem with previous studies is that if the consumption income ratio is adjusted in an asymmetric or non-linear way, then conventional unit root tests suffer

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<sup>1</sup> Readers are referred to Parker (2010), Baykara and Telatar (2012) and Cerrato et al. (2013) for a theoretical review of various consumption theories and their empirical implications in regard to the stationarity of the consumption-income ratio.

<sup>2</sup> Drobny and Hall (1989), Molana (1991), Hall and Patterson (1992) and Horioka (1997) are examples of studies where it is concluded that the APC is characterized by non-stationarity. Ungern-Sternberg (1986) and King et al. (1991) are by contrast, examples where it is concluded that the APC is characterized by stationarity.

from a loss of power that may lead to the acceptance of non-stationarity when the consumption-income ratio is actually stationary. Therefore, a growing body of researches (see, for example, Cook, 2002; Tsionas and Christopoulos, 2002; Baykara and Telatar, 2012; Elmi and Ranjbar, 2013) has turned their attention to the adoption of more sophisticated non-linear models to test the stationarity of the consumption-income ratio. The empirical evidence from this line of research indicates that, by taking the non-linear property into account, the APCs of the OECD countries are no longer in violation of the stationarity. For the benefit of readers, we summarize the recent contributions to this issue in Table 1.

**Table 1: Summary of recent contributions on stationarity of the consumption-income ratio**

Studies	Countries and samples covered	Methodology	Stationarity of the consumption-income ratio
Sarantis and Stewart (1999)	20 OECD countries, 1955–1994	Panel unit root tests of Im et al. (2003) and Taylor and Sarno (1998)	Non-stationarity is not rejected for these OECD countries
Tsionas and Christopoulos (2002)	14 EU countries, 1960--1999	Panel unit root test and asymmetric unit root test	Non-stationarity is not rejected for these EU countries
Cook (2002)	20 OECD countries, 1955--1994	Enders and Granger (1998) MTAR unit root test	Non-stationarity is not rejected for 18 of 20 OECD countries
Cook (2003)	UK, 1955--2001	Weighted symmetric DF test and recursive mean adjusted DF test	Non-stationarity is not rejected for the UK
Cook (2005)	20 OECD countries, 1955--1994	Lee and Strazicich (2003) minimum LM unit root test with two structural breaks	Non-stationarity is not rejected for these OECD countries
Romero- Avila (2008)	23 OECD countries, 1960--2005	Panel unit root tests of Smith et al. (2004) and Hadri (2000)	Non-stationarity is not rejected for these OECD countries
Romero- Avila (2009)	23 OECD countries, 1960--2005	Panel unit root tests with and without structural breaks	Regime-wise stationarity is rejected for these OECD countries by applying panel unit root test with multiple breaks
Gomes (2009)	10 South American countries, 1951--2003	Lee and Strazicich (2003) minimum LM unit root test with two structural breaks	Non-stationarity is rejected for these countries
Liao et al. (2010)	22 OECD countries, 1970--2006	Breuer et al. (2002) SURADF panel unit root test	22 out of 24 series are stationary processes
Fallahi (2012)	23 OECD countries, 1950--2007	Hansen's (1999) grid bootstrapping confidence intervals	Non-stationarity is not rejected for most of the countries
Baykara and Telatar (2012)	14 transition economies, 1994--2009	Kapetanios et al. (2003) and Sollis (2009) ESTAR-type unit root tests	Non-stationarity is rejected for these countries
Elmi and Ranjbar (2013)	16 OECD countries, 1960--2010	Becker et al. (2006) fourier stationary test	Stationarity is not rejected for 12 of 16 OECD countries
Cerrato et al. (2013)	24 OECD and 33 non-OECD countries, 1951--2003	Panel unit root tests of Cerrato et al. (2009) and Pesaran (2007)	Non-stationarity is not rejected for 83% of OECD and 74% of non-OECD countries
Gozgor (2013)	11 Central and Eastern European (CEE) countries, 1997--2012	Panel unit root tests Maddala and Wu (1999) and Pesaran (2007)	Mean-reversion in the consumption-income ratio for 9 of 11 CEE economies

There are at least two channels that make the consumption-income series become a non-linear process. The first rationale for incorporating possible asymmetry in the adjustment of the consumption-income ratio is the existence of durable consumption that can be addressed as a direct source of non-linear and asymmetric adjustment for consumption to changes in income. Indeed, any durable component in consumption would likely be particularly slow to adjust (Cerrato et al., 2013). Second, the available empirical evidence suggests that various business cycle indicators exhibit asymmetric behavior (see, for example, Enders and Siklos, 2001). Given that the consumption is influenced by aggregate income or business cycle movements via the aggregate consumption function, it is reasonable to assume that the business cycle asymmetries could possibly translate into consumption.

The aim of this study is to re-examine whether or not the consumption-income ratios of the OECD countries are non-stationary processes. An important implication of the standard unit root tests is the implicit assumption that the adjustment process is symmetric. Indeed, if the true adjustment process is asymmetric, then the restrictive symmetric adjustment implicitly assumed is indicative of model misspecification. Moreover, Cerrato et al. (2013, p. 106) highlight that "a test allowing for non-linear adjustment towards a changing target APC may be more appropriate than assuming that all countries are subject to intercept and slope shifts in a single period." As such, we employ Chen and Xie's (2015) test that could encompass asymmetry and non-linear trend under the alternative hypothesis. That is, we test for the null hypothesis of a unit root against the alternative hypothesis that encompasses a structural break and asymmetry at the same time.

Chen and Xie (2015) propose a two-step testing strategy: first estimating non linear trend and then applying various non-linear unit root test with no deterministic component. The idea of this procedure is in line with Leybourne et al. (1998), Sollis (2004) and Cook and Vougas (2009). In the first step, they employ logistic smooth transition models proposed by Leybourne et al. (1998) to model the non-linearity that stems from a structural break.<sup>24</sup> These models permit the possibility of a smooth transition between two different trend paths over time.<sup>25</sup> In the second step, they consider a possibility that the adjustment speed is non-linear (i.e., *size non-linearity*) and follows an exponential smooth transition autoregressive (ESTAR) process.<sup>26</sup> In this paper, we consider a variety of the ESTAR-type unit root tests, i.e., Kapetanios et al. (2003) and Park and Shintani (2012), to the residuals from the first step. The ESTAR function implies that that the speed of mean reversion is faster when the transition variable is sufficiently far away from zero. In other words, mean reversion will be faster when the consumption-income ratio is far from its equilibrium value determined by the exponential function, whereas it behaves as a unit root process when it is close to it.

In addition, in order to take account of the possibility of an asymmetric adjustment of the APC, first, we also adopt the threshold autoregressive (hereafter TAR) and the momentum threshold autoregressive (hereafter MTAR) unit root tests, as proposed by Enders and Granger

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<sup>24</sup> Leybourne and Mizen (1999) point out that "when considering aggregate behavior, the time path of structural changes in economic series is likely to be better captured by a model whose deterministic component permits gradual rather than instantaneous adjustment."

<sup>25</sup> As noted by Davidson et al. (1978), any durable component in consumption would likely be particularly slow to adjust. As a result, changes in APC would likely occur over several time periods rather than in a single shot. Thus, it is worth arguing that a test allowing non-linear adjustment towards a changing APC target is more appropriate than assuming that all countries are subject to intercept and slope shifts in a single period.

<sup>26</sup> This non-linear behavior implies that there is a central regime where the series behave as a unit root whereas for values outside the central regime, the variable tends to revert to the equilibrium.

(1998), to characterize the so-called *sign asymmetry* in this study. These unit root tests that encompass the non-linear trend and asymmetric adjustment are labeled LNV-TAR and LNV-MTAR models, which is championed Sollis (2004) and Cook and Vougas (2009), respectively.

As compared to the literature, the contributions of this study are as follows. Previous studies (e.g., Cook, 2005; Gomes, 2009; Elmi and Ranjbar, 2013) have shed light on the importance of recognizing the possibility of a structural shift in testing for the null hypothesis of a unit root for the consumption-income ratio. We take this possibility into consideration by employing the non-linear unit root tests proposed by Sollis (2004), Cook and Vougas (2009) and Chen and Xie (2015) approaches. As noted by Davidson et al. (1978), any durable component in consumption would likely be particularly slow to adjust. As a result, changes in the APC would likely occur over several time periods rather than in a single shot. Thus, it is worth arguing that a test allowing non-linear adjustment towards a changing APC target is more appropriate than assuming that all countries are subject to intercept and slope shifts in a single period. From applied economic point of view, the application of the non-linear model overcomes the weakness of the traditional linear unit root test in detecting the stationarity of the APC. It allows us to draw conclusions about the validity of the theoretical foundation for the APC in the long run.

The remainder of this paper is organized as follows. Section 2 outlines the statistical methods used for testing for a unit root under assumptions of a structural break and non-linearity. Section 3 discusses the data used and the empirical results and compares our results with those of the extant literature. Finally, Section 4 concludes.

## 2 Econometric Methodology

### 2.1 Structural Break nonlinearity with LSTR Unit Root Test

As mentioned in the introduction, one of deviations from the fundamental equilibrium is that the consumption-income ratio has undergone a structural break. In other words, non-linearity may affect a variable in the form of structural changes in the deterministic components. That is, a broken time trend is a particular case of a non-linear time trend. Leybourne, Newbold and Vougas (1998) (hereafter LNV) develop a unit root test against the alternative hypothesis of stationarity around a logistic smooth transition non-linear trend. It is appealing as it permits structural shifts to occur gradually over time.

Leybourne et al. (1998) consider three models:

$$\text{Model A } y_t = \alpha_1 + \alpha_2 S_t(\gamma, \tau) + v_t, \quad (1)$$

$$\text{Model B } y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t(\gamma, \tau) + v_t, \quad (2)$$

$$\text{Model C } y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t(\gamma, \tau) + \beta_2 t S_t(\gamma, \tau) + v_t, \quad (3)$$

where  $v_t$  is a zero mean  $I(0)$  process,  $S_t(\gamma, \tau)$  is the logistic smooth transition function:

$$S_t(\gamma, \tau) = [1 + \exp\{-\gamma(t - \tau T)\}]^{-1} \quad (4)$$

The parameter  $\tau$  determines the timing of the transition midpoint. Since  $\gamma > 0$ , we have

$t \rightarrow -\infty, S_{-\infty}(\gamma, \tau) = 0, t \rightarrow +\infty, S_{-\infty}(\gamma, \tau) = 1,$  and  $S_{\tau}(\gamma, \tau) = 0.5$ . The speed of transition is determined by the parameter  $\gamma$ . If  $v_t$  is a zero-mean  $I(0)$  process, then in Model A  $y_t$  is stationary around a mean which changes from the initial value  $\alpha_1$  to the final value  $\alpha_1 + \alpha_2$ . Model B is similar, with the intercept changing from  $\alpha_1$  to  $\alpha_1 + \alpha_2$ , but it allows for a fixed slope term. In Model C, in addition to the change in intercept from  $\alpha_1$  to  $\alpha_1 + \alpha_2$ , the slope also changes simultaneously, and with the same speed of transition, from  $\beta_1$  to  $\beta_1 + \beta_2$ .

The null hypothesis and alternative hypothesis are as follows:

$$H_0 \quad v_t = \mu_t, \quad \mu_t = \mu_{t-1} + \varepsilon_t, \tag{5}$$

where  $\varepsilon_t$  is assumed to be a stationary process with zero mean. The test statistics are then calculated via a two step procedure. The procedure is implemented as follows:

Step 1: The first step involves estimating non-linear deterministic component in models (1)–(3) by the non-linear least square. We then compute the residuals

$$\text{Model A} \quad \hat{v}_t = y_t - \hat{\alpha}_1 - \hat{\alpha}_2 S_t(\hat{\gamma}, \hat{\tau}), \tag{6}$$

$$\text{Model B} \quad \hat{v}_t = y_t - \hat{\alpha}_1 - \hat{\beta}_1 t + \hat{\alpha}_2 S_t(\hat{\gamma}, \hat{\tau}), \tag{7}$$

$$\text{Model C} \quad \hat{v}_t = y_t - \hat{\alpha}_1 + \hat{\beta}_1 t + \hat{\alpha}_2 S_t(\hat{\gamma}, \hat{\tau}) + \hat{\beta}_2 t S_t(\hat{\gamma}, \hat{\tau}), \tag{8}$$

Step 2. In the second step we test for a unit root on the residuals of step one.

$$\Delta \hat{v}_t = \rho \hat{v}_{t-1} + \sum_{i=1}^k \delta_i \Delta \hat{v}_{t-i} + \hat{\eta}_t \tag{9}$$

where  $\gamma > 0$  and the lagged terms of  $\Delta \hat{v}_{t-i}$  is added to regressions to assure the error term  $\hat{\eta}_t$  is a white noise. Eq. (9) is a standard ADF regression that assumes linear adjustment toward equilibrium, which is proposed by Leybourne et al. (1998). They suggest a two-step testing strategy, first estimating Eqs. (1)–(4) by the non-linear least squares, and then applying an ADF test with no deterministic component to Eq. (9). The statistics are labeled  $s_\alpha, s_{\alpha(\beta)}$  and  $s_{\alpha\beta}$  corresponding to Models A to C, respectively. We call this test the LNV-ADF test.

## 2.2 Test for a structural break and sign asymmetry with the LNV-MTAR Model

In order to consider sign asymmetry and a structural break at the same time, Cook and Vougas (2009) combine the ideas of Enders and Granger (1998) and Leybourne et al. (1998) develop a new test for the null hypothesis of a unit root, that under the alternative hypothesis not only allows for the possibility of a regime shift between two different trend paths over time, but also permits a structural break to occur gradually over time instead of instantaneously. That is, we test for a unit root on the residuals from Eqs. (6) to (8) as follows:

$$\Delta \hat{v}_t = M_t \rho_1 \hat{v}_{t-1} + (1 - M_t) \rho_2 \hat{v}_{t-1} + \sum_{i=1}^k \delta_i \Delta \hat{v}_{t-i} + \hat{\eta}_t \quad (10)$$

where  $M_t$  is the Heaviside indicator function,

$$M_t = \begin{cases} 1, & \text{if } \Delta \hat{v}_{t-1} \geq 0, \\ 0, & \text{if } \Delta \hat{v}_{t-1} < 0 \end{cases} \quad (11)$$

again,  $\hat{v}_t$  is the residual from the first step by using the non-linear least squares for Eqs (6)–(8). The combination of Eqs (6)–(8) and Eq. (10) is called the LNV-MTAR test.<sup>27</sup> If the transition functions (1)–(3) are replaced by Leybourne et al. (1998), then this test is called LNV-MTAR proposed by Cook and Vougas (2009).

In this new test, if the null hypothesis of  $H_0 : \rho_1 = \rho_2 = 0$  cannot be rejected in Eq. (10), then  $\hat{v}_t$  and therefore  $y_t$  contains a unit root. The statistics are referred to as  $F_{\alpha}^*$ ,  $F_{\alpha(\beta)}^*$  and  $F_{\alpha\beta}^*$  correspond to Models A to C, respectively. If  $H_0 : \rho_1 = \rho_2 = 0$  is rejected and  $\rho_1 = \rho_2 < 0$  hold, then  $\hat{v}_t$  ( $y_t$ ) is a stationary LNV-MTAR process with symmetric adjustment. If  $H_0 : \rho_1 = \rho_2 = 0$  is rejected and  $\rho_1 < 0$ ,  $\rho_2 < 0$ ,  $\rho_1 \neq \rho_2$  holds, then  $\hat{v}_t$  ( $y_t$ ) is a stationary LNV-MTAR process displaying asymmetric adjustment.

### 2.3 Test for a structural break and size non-linearity with the LNV-ESTAR approaches

In order to take account of the possibility of a smooth break and asymmetric speed of adjustment towards equilibrium simultaneously, we assume that the adjustment speed is non-linear and follows an exponential smooth transition autoregressive (ESTAR) process as shown in Eqs. (12) to (13). This implies that the consumption-income may be a unit root process for a given threshold of values (inner regime), but a stationary process when the consumption-income ratio reaches the outer regime.

We consider the following three non-linear models:

$$\Delta \hat{v}_{t-1} = \psi \hat{v}_{t-1} [1 - \exp(-\gamma \hat{v}_{t-1}^2)] + \sum_{i=1}^k \delta_i \Delta \hat{v}_{t-i} + \hat{\eta}_t \quad (12)$$

$$\Delta \hat{v}_{t-1} = \lambda \hat{v}_{t-1}^3 + \sum_{i=1}^k \delta_i \Delta \hat{v}_{t-i} + \hat{\eta}_t \quad (13)$$

$$\Delta \hat{v}_{t-1} = \lambda_1 \hat{v}_{t-1}^3 + \lambda_2 \hat{v}_{t-1}^4 + \sum_{i=1}^k \delta_i \Delta \hat{v}_{t-i} + \hat{\eta}_t \quad (14)$$

<sup>27</sup> If we replace equation (11) with

$$I_t = \begin{cases} 1, & \text{if } \hat{v}_{t-1} \geq 0, \\ 0, & \text{if } \hat{v}_{t-1} < 0 \end{cases}$$

then it is called the logistic smooth transition TAR (LNV-TAR) model proposed by Sollis (2004).



where  $\lambda > 0$  and the lagged terms of  $\Delta\hat{v}_{t-i}$  is added to regressions to assure the error term  $\hat{\eta}_t$  is a white noise. These correspond to the unit root tests developed by Park and Shintani (2012) [Eq. (12)] and Kapetanios et al. (2003) [Eq. (13)]. All models allow for testing for a unit root in the original series after removing the smooth breaks in the deterministic component.

Eqs (12) to (13) allow, in addition to temporary breaks, testing for a unit root against a non-linear alternative. The transition parameter  $\gamma$  determines the speed of transition between two extreme regimes. Park and Shintani (2012) develop a unit root test against general transition AR model that uses  $v_{t-1}$  as transition variable instead of  $\Delta v_{t-1}$ . The test can be implemented as follows. Let  $\Gamma_n$  denote a random sequence of parameter spaces given for each n as functions of the sample  $(v_1, v_2, \dots, v_n)$ . For each  $\gamma \in \Gamma_n$ , one obtains the t-statistic for  $\psi$  in (12),

$$T_n(\gamma) = \frac{\hat{\psi}(\gamma)}{se(\hat{\psi}(\gamma))} \tag{15}$$

where  $\hat{\psi}(\gamma)$  is the least squares estimate and  $se(\hat{\psi}(\gamma))$  is the corresponding standard error. The inf-t test is then defined as

$$T_n = \inf_{\gamma \in \Gamma_n} T_n(\theta) \tag{16}$$

which is the infimum of t-ratios in model (12) taken over all possible values of  $\gamma \in \Gamma_n$ . We label this test the LNV-inf-PS test.

Kapetanios et al. (2003) develop a test for the null hypothesis of a unit root against an alternative of a non-linear but globally stationary smooth transition autoregressive process. The Kapetanios et al. (2003) unit root test is in fact a linearized version of the Kilic (2011) test that uses, like Park and Shintani (2012),  $v_{t-1}$  as transition variable. The null hypothesis to be tested with Eq. (13) is  $H_0 : \lambda = 0$  (unit root in outer regime) against the alternative of  $H_1 : \lambda < 0$  (stationarity in outer regime). Specifically, the test is obtained with the following t-statistic

$$t = \frac{\hat{\lambda}}{se(\hat{\lambda})} \tag{17}$$

We refer to this test as the KSS non-linear augmented Dickey-Fuller test and label it the LNV-KSS test.

Finally, the speed of mean reversion may actually depend not only on the absolute deviation from the equilibrium, but also upon the sign of the shock. Intuitively, it makes sense to think that a negative shock on the consumption-income ratio may be more difficult to tackle than a positive shock. Sollis (2009) proposes testing the unit root hypothesis using an extended version of the ESTAR model that allows for symmetric or asymmetric non-linear adjustment under the

alternative hypothesis. The suggestion of Sollis (2009) is to simplify the model further by taking a Taylor expansion of the logistic function and the resulting model is Eq. (14).<sup>28</sup> The null hypothesis is  $H_0 : \lambda_1 = \lambda_2 = 0$  in the auxiliary model (14). A feature of the proposed asymmetric ESTAR (AESTAR) model is that if the unit root hypothesis has been rejected against the alternative of stationary symmetric or asymmetric ESTAR non-linearity, the null hypothesis of symmetric ESTAR non-linearity can then be tested against the alternative of AESTAR non-linearity using the auxiliary model by testing  $H_0 : \lambda_2 = 0$  against  $H_1 : \lambda_2 \neq 0$  with a standard F-test. To clarify, it can be seen from model (14) that if  $\lambda_2 = 0$ , the AESTAR auxiliary model collapses to the ESTAR auxiliary model of Kapetanios et al. (2003). For standard F critical values to be applicable for this test, the parameter  $\lambda_1$  should be negative ( $\lambda_1 < 0$ ), so that under the null being tested the series is stationary. We label this test the LNV-Sollis-AESTAR test. Please note that this test encompasses smooth break, size non-linearity and sign asymmetry under the alternative hypothesis at the same time. While the LNV-inf-PS and LNV-KSS consider smooth break and size non-linearity under the alternative in testing for the null hypothesis of a unit root.

The critical values for Eqs. (9) [LNV-ADF] and (10) [LNV-MTAR] can be extracted from Leybourne et al. (1998) and Cook and Vougas (2009), respectively. However, in estimating the models in Eqs. (12)–(14) [LNV-inf-PS, LNV-KSS and LNV-Sollis-AESTAR], the critical values are available in Chen and Xie (2015).

### 3 Data and Results

#### 3.1 Data description and basic statistics

The data include annual observations of the consumption-income ratios. We focus on twenty OECD countries, namely Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, the Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, the UK and the US in our empirical study. The sample periods are dependent on the availability of the data, which cover the period from 1950 to 2010. All data are obtained from Penn World Table 7.1.

Some descriptive statistics of the consumption-income ratios are outlined in Table 2, which details the first four moments of each series and presents tests for normality and serial correlation. Several interesting facts are observed from Table 2. First, with the exception of Belgium, Ireland, New Zealand and Norway, the coefficients of skewness of all of the series are positive, implying that the consumption-income ratios are flatter to the right compared to the normal distribution. The coefficients of excess kurtosis for these ratios are above 0 except for Iceland, indicating that the empirical distributions of these samples have fat tails. The coefficients of skewness and excess kurtosis reveal non-normality in the data with the exception of Austria, Belgium, Denmark, Greece, Ireland, the Netherlands and Norway. This is confirmed by the Jarque-Bera normality test as shown in Table 2. Second, the Ljung-Box Q-statistics, LB(24), denote significant autocorrelations for most of the series. We also report a standard ARCH test for the consumption-income ratios. The test results indicate that a significant ARCH effect does not exist for most of the consumption-income ratios, with the exception of the Netherlands, New Zealand, Norway and Spain.

<sup>28</sup> Readers are referred to Sollis (2009) for detailed discussion.

**Table 2: Summary statistics**

	Mean	S.D.	SK	EK	JB	LB(24)	ARCH(4)
Austria	- 0.001	0.101	0.417	0.229	1.847	56.324**	1.841
Belgium	- 0.002	0.011	-0.185	0.719	1.911	38.831**	0.181
Canada	- 0.001	0.009	0.631	0.663	5.089*	55.965**	1.314
Denmark	- 0.001	0.010	0.269	0.698	1.944	38.412**	0.122
Finland	- 0.001	0.015	0.788	1.967	15.890**	22.713	0.504
France	0.001	0.007	1.110	4.206	56.579***	36.231**	0.444
Greece	0.001	0.020	0.213	0.108	0.003	55.981**	0.653
Iceland	- 0.001	0.020	0.088	-0.396	0.470	49.573**	0.309
Ireland	- 0.003	0.015	-0.829	1.095	9.875**	43.589**	0.290
Italy	- 0.003	0.011	0.448	1.026	4.645*	35.493*	0.275
Japan	- 0.001	0.012	0.907	2.017	18.406**	21.404	0.340
Luxembourg	0.001	0.019	1.331	3.757	53.036***	27.357	0.205
Netherlands	- 0.001	0.009	0.305	0.442	1.420	50.381**	3.300**
New Zealand	< 0.001	0.014	-0.078	2.893	20.992**	59.156**	2.091*
Norway	- 0.001	0.009	-0.015	0.345	0.301	30.019*	2.722**
Spain	- 0.001	0.010	0.056	3.675	33.806**	22.417	6.447**
Sweden	- 0.001	0.010	0.727	2.662	23.012**	27.955	0.360
Switzerland	- 0.001	0.014	0.365	2.243	19.913**	27.350	0.272
the UK	< - 0.001	0.008	-0.597	1.037	6.260**	37.731**	0.535
the US	0.001	0.008	0.012	1.715	7.357**	27.226	0.289

(1) \*, \*\*, \*\*\* denote significance at the 10%, 5% and 1%, respectively. (2) Mean and S.D. refer to the mean and standard deviation, respectively. (3) SK is the skewness coefficient. (4) EK is the excess kurtosis coefficient. (5) JB is the Jarque-Bera statistic. (6) LB(24) is the Ljung-Box Q statistic calculated with twenty-four lags. (7) ARCH(4) is the ARCH test calculated with four lags on raw returns. (8) Term ' $< 0.001$ ' indicates that the number is less than 0.001.

As a preliminary analysis, we apply a battery of linear unit root tests to determine the order of integration of the consumption-income ratio.<sup>29</sup> We consider the Augmented Dickey-Fuller (ADF) test, as well as the ADF-GLS test of Elliott et al. (1996) in this study. Vougas (2007) highlights the usefulness of the Schmidt and Phillips (1992) (SP hereafter) unit root test in practice. Therefore, we also employ it in this study. These authors propose some modifications of existing linear unit root tests in order to improve their power and size. For the ADF and ADF-GLS tests, an auxiliary regression is run with an intercept and a time trend. To select the lag length ( $k$ ) we use the 't-sig' approach proposed by Hall (1994). That is, the number of lags is chosen for which the last included lag has a marginal significance level below the 10% level.

The results of applying these tests are reported in Table 3. We find that, with the exception of Belgium, Canada, France, Greece, Ireland, Switzerland and the US, the null hypothesis of a unit root cannot be rejected at the 10% significance level for the ADF statistics. Based on the well-known low power problem of the ADF test, we turn our attention to other statistics. The SP test (see Schmidt and Phillips, 1992), with parametric correction, cannot reject the unit root hypothesis with both linear and quadratic trend at the five percent significance level with the exception of New Zealand and the US.<sup>30</sup> The results from the DF-GLS (see Elliott et al., 1996)

<sup>29</sup> An important advantage of the stationarity test is that it does not involve the estimation of an unknown cointegrating parameter.

<sup>30</sup> The terms SP(1) and SP(2) tests, denote the Schmidt-Phillips  $\tau$  tests with the linear and quadratic trend,

echo the results of the SP(1) and SP(2), suggesting that the consumption-income ratios for these countries are non-stationary processes. Based on the linear unit root test results, the presence of a unit root in the consumption-income ratio is not in accordance with a constant APC in the long run.

As Perron (1989) pointed out, in the presence of a structural break, the power to reject a unit root decreases if the stationary alternative is true and the structural break is ignored. To address this, we use Zivot and Andrews' (1992) sequential one trend break model and Lumsdaine and Papell's (1997) two trend breaks model to investigate the order of the empirical variables. We use the 't-sig' approach proposed by Hall (1994) to select the lag length ( $k$ ). We set  $k_{\max} = 12$  and use the approximate 10% asymptotic critical value of 1.60 to determine the significance of the t-statistic on the last lag. We use the 'trimming region'  $[0.10T, 0.90T]$  and select the break point endogenously by choosing the value of the break that maximizes the ADF t-statistic. We report the results in Table 3. With the exceptions of Austria, Luxembourg, Switzerland and the UK, the null hypothesis of a unit root cannot be rejected at the conventional significance level. The results of the LP test suggest that the null hypothesis of a unit root cannot be rejected at the 5% significance level for all of the consumption-income ratios. These findings fully echo those obtained from the SP and DF-GLS linear unit root tests.

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respectively.

**Table 3: Results of the linear unit root tests**

	Linear Trend			Quadratic trend and breaks tests		
	ADF	SP(1)	DF-GLS	SP(2)	ZA, Model C	LP, Model C
Austria	-2.915	-2.819	-2.870	-3.554	-5.228**	-4.550
Belgium	-3.502**	-2.431	-2.380	-2.168	-4.647	-4.596
Canada	-3.529**	-1.743	-1.367	-2.185	-3.941	-4.887
Denmark	-2.581	-1.934	-1.481	-2.038	-2.955	-4.688
Finland	-2.960	-2.968	-2.686	-2.423	-3.743	-4.605
France	-3.296*	-2.055	-1.771	-2.137	-4.224	-4.138
Greece	-3.238*	-1.430	-1.221	-1.789	-2.915	-5.396
Iceland	-1.095	-1.630	-2.225	-2.607	-4.461	-3.662
Ireland	-3.454**	-2.341	-2.340	-2.387	-3.576	-4.541
Italy	-3.107	-2.073	-1.873	-1.860	-4.404	-5.800
Japan	-0.923	-1.519	-1.106	-2.656	-3.678	-5.456
Luxembourg	-2.341	-1.370	-1.203	-2.984	-6.342***	-5.834
Netherlands	-2.292	-2.115	-2.259	-2.246	-4.653	-5.363
New Zealand	-2.360	-2.918	-4.034**	-4.985**	-4.047	-4.424
Norway	0.541	-1.498	-0.316	-1.657	-3.740	-4.156
Spain	-2.487	-1.875	-1.497	-2.570	-4.804	-5.377
Sweden	-1.771	-1.829	-1.812	-2.019	-4.288	-4.354
Switzerland	-3.545**	-1.350	-1.777	-1.918	-5.820***	-5.039
the UK	-2.853	-1.465	-1.551	-3.700	-2.127**	-5.610
the US	-3.484**	-3.805***	-3.395**	-3.136	-4.366	-5.737

(1) \*, \*\*, \*\*\* denote significance at the 10%, 5% and 1%, respectively. (2) ADF, SP(1) and DF-GLS denote the augmented Dickey-Fuller test, Schmidt-Phillips  $\tau$  test with linear trend and Elliott et al. (1996) DF-GLS test, respectively. (3) SP(2), ZA and LP denote the Schmidt-Phillips  $\tau$  test with quadratic trend, Zivot and Andrews (1992) and Lumsdaine and Papell (1997) tests, respectively. (4) The 5% critical values for the ADF, SP(1) and DF-GLS tests are -3.43, -3.04 and -2.89, respectively. (5) The 5% critical values for the SP(2), ZA and LP tests are -3.55, -5.08 and -6.75, respectively.

### 3.2 Results of the LNV-ADF, LNV-TAR and LNV-MTAR approaches

Bierens (1997) and Leybourne et al. (1998) emphasize that non-linearity may occur in the form of structural changes in the deterministic components. That is, a broken time trend is a particular case of a non-linear time trend. In this study, for example, there is an obvious shift in the trend of the consumption-income ratio for Italy circa 1960 and Spain circa 1970 (see Figures 1 and 2). Of particular importance, the shift seems to be smooth rather than abrupt. In order to take the possibility of the smooth transition non-linear trends into consideration, we use the logistic smooth transition ADF test, proposed by Leybourne et al. (1998) in this study.

Following the two-step procedure described in the previous section, we first fitted the logistic smooth transition model to the consumption-income ratio. Figures 1 and 2 present the times series plots of the consumption-income ratios (black line) and the estimated logistic smooth transition functions (blue line) for these twenty OECD countries, respectively.<sup>31</sup> It is found that the estimated logistic smooth transition trends are close to the raw data.

<sup>31</sup> The detailed estimation results of the logistic smooth transition model, i.e., Eqs (1)--(4), are available from the author upon request.

# Nonlinear mean reversion in the consumption-income ratio: New evidence from the OECD countries

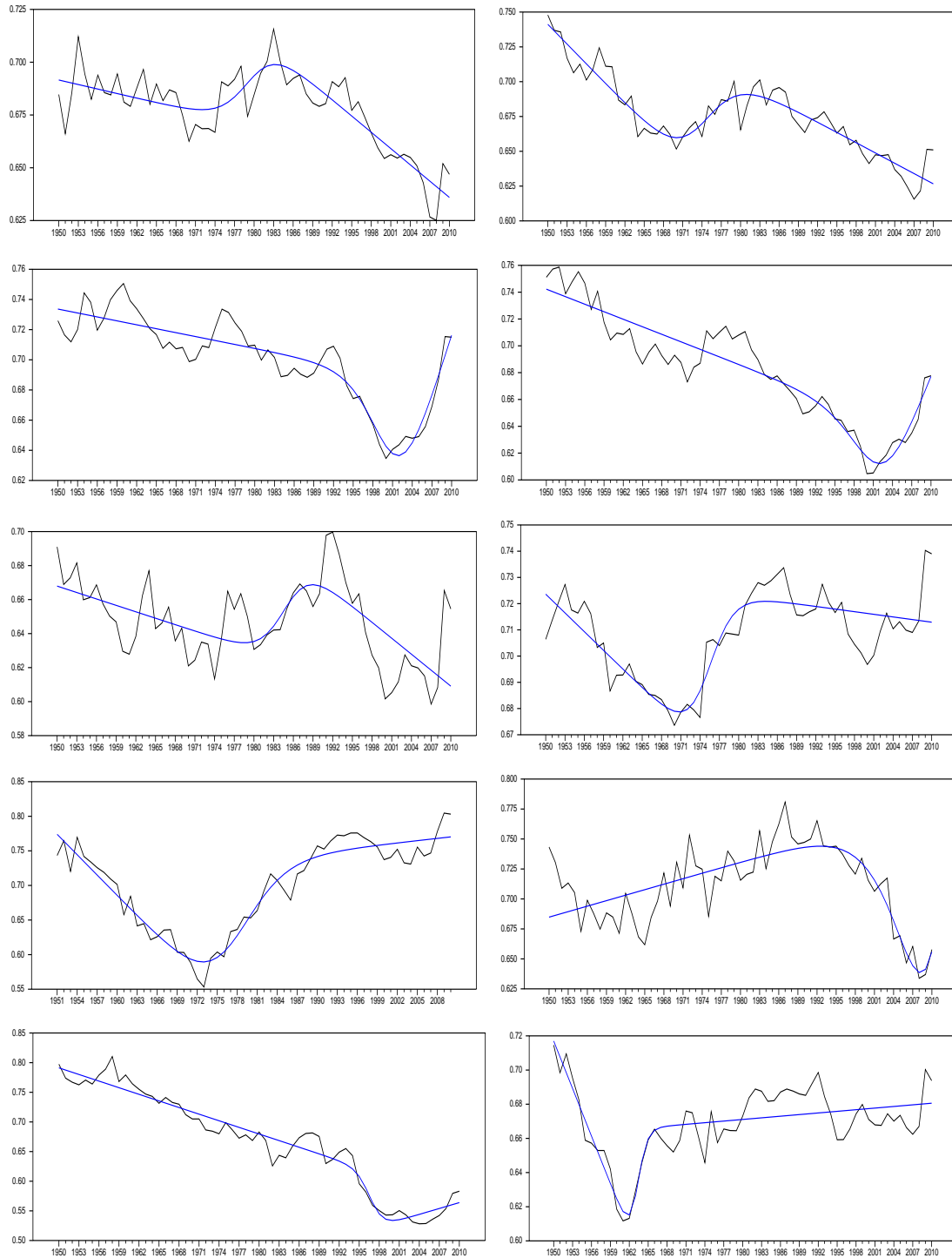


Figure 1: Consumption-income ratio (black line) and the fitted logistic smooth transition function (blue line) for Model C. The order of countries from left to right are Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Ireland and Italy.

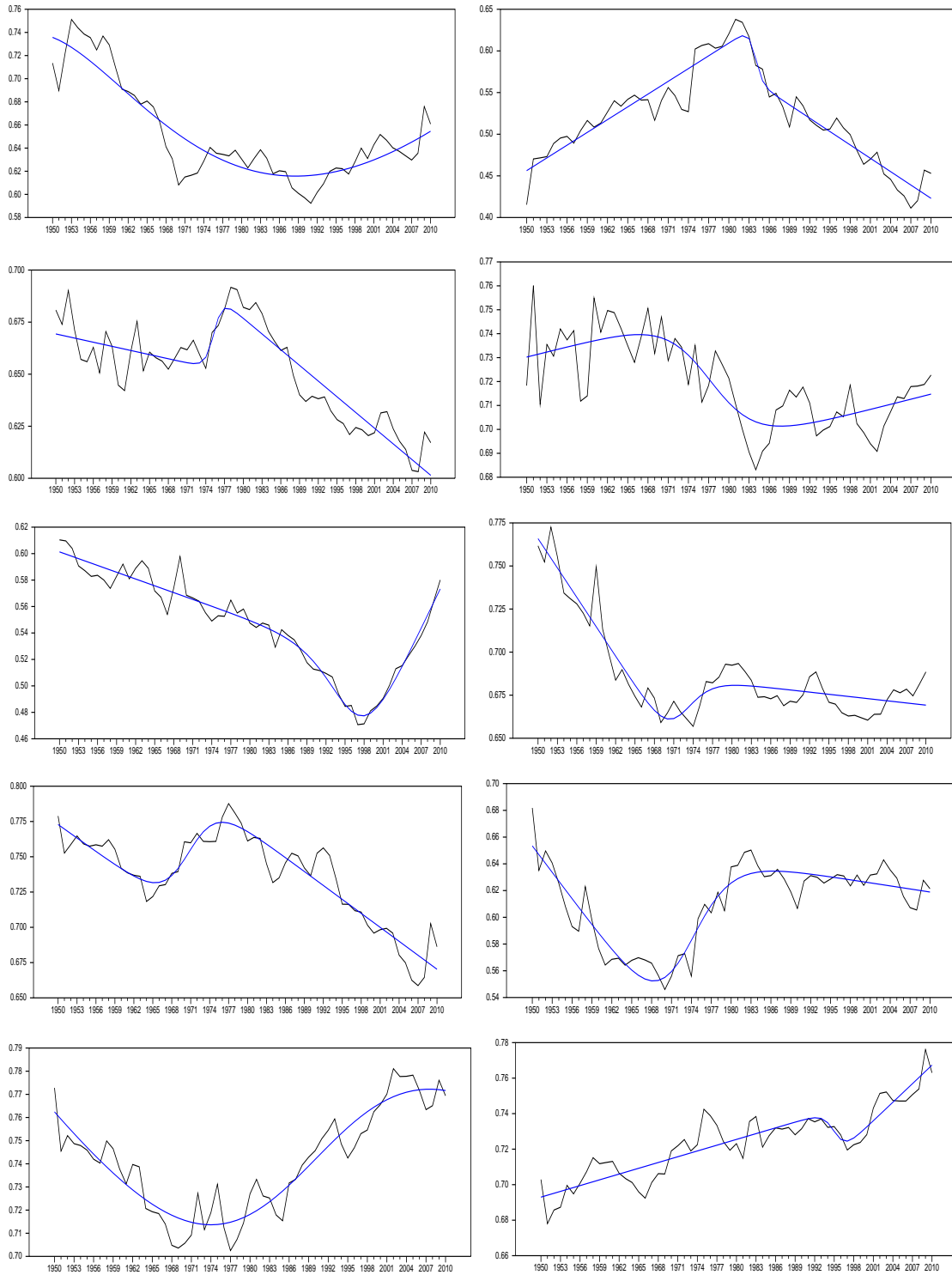


Figure 2: Consumption-income ratio (black line) and the fitted logistic smooth transition function (blue line) for Model C. The order of countries from left to right are Japan, Luxembourg, Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, the UK and the US.

We then proceed to Step 2 of our testing procedure and obtain the residuals from the logistic smooth function and apply the four unit root tests proposed above. The first one is the LNV-ADF test where we assume that mean reversion follows a linear process. The optimal lag order of the ADF regression is selected by the Bayesian information criterion (BIC). We summarize the test results of the LNV-ADF statistic in Table 4. Based on the  $s_{\alpha}$ ,  $s_{\alpha(\beta)}$  and  $s_{\alpha\beta}$  statistics, it is found that for nine of the twenty countries, the null hypothesis of a unit root is rejected at least at the 10% significance level or better, indicating that the consumption-income ratios of these countries (i.e., Austria, Belgium, Canada, Iceland, New Zealand, Norway, Sweden, Switzerland and the UK) are stationary processes around a logistic smooth transition non-linear trend. That is, although the break alone can account for rejection of the unit root null, the evidence in favor of stationary APC is not sufficiently strong. Next, we consider to test for a unit root against the alternative hypothesis with the smooth transition non-linear trend with threshold adjustment LNV-TAR and LNV-MTAR models, championed by Sollis (2004) and Cook and Vougas (2009), respectively, in this study. These approaches permit structural shifts to occur gradually over time instead of instantaneously.



**Table 4: Results of the LNV-ADF unit root test**

	LNV-ADF		
	Model A	Model B	Model C
Austria	- 3.632	- 3.705	- 4.819**
Belgium	- 2.111	- 3.374	- 5.083**
Canada	- 4.045*	- 2.894	- 3.926
Denmark	- 2.031	- 2.482	- 2.975
Finland	- 3.526	- 3.775	- 3.843
France	- 2.558	- 2.105	- 2.992
Greece	- 2.177	- 1.818	- 3.198
Iceland	- 2.075	- 4.564**	- 3.362
Ireland	- 2.851	- 2.849	- 3.435
Italy	- 2.227	- 3.093	- 3.542
Japan	- 2.629	- 3.026	- 3.027
Luxembourg	- 2.174	- 3.451	- 3.915
Netherlands	- 2.137	- 3.618	- 3.624
New Zealand	- 4.547**	- 5.256**	- 5.256**
Norway	- 0.541	- 3.225	- 5.537**
Spain	- 3.275	- 3.287	- 3.631
Sweden	- 3.226	- 4.634**	- 4.633**
Switzerland	- 2.972	- 2.711	- 4.867**
the UK	- 2.923	- 4.465*	- 4.753*
the US	- 2.923	- 3.475	- 3.823
10% cv	- 3.851	- 4.337	- 4.572
5% cv	- 4.161	- 4.629	- 4.867
1% cv	- 4.761	- 5.201	- 5.435

(1) \*, \*\*, \*\*\* denote significance at the 10%, 5% and 1% levels, respectively. (2) LNV-ADF denotes the non-linear unit root test proposed by Leybourne et al. (1998). (3) The critical values for the LNV-ADF statistics are obtained from Leybourne et al. (1998).

The LNV-TAR specification is examined by first testing the null hypothesis of a unit root, i.e.,  $\rho_1 = \rho_2 = 0$ , in equation (10) and comparing the appropriate critical values from Sollis (2004). The results of applying the LNV-TAR test for Model B as well as Model C of the consumption-income ratios of these OECD countries are reported in the left and right panels of Table 5, respectively. From the results of the  $F_{\alpha(\beta)}$  and  $F_{\alpha\beta}$  as shown in Table 5, for Austria, Luxembourg, Norway, Sweden, Switzerland and the UK, the null hypothesis of a unit root is rejected at the 5 percent significance level or better. The results imply that the consumption-income ratios are non-linear trend stationary processes for these countries. The rejections obtained from  $F_{\alpha(\beta)}$  and  $F_{\alpha\beta}$  (Models B and C of the LNV-TAR model) reveal that the hypothesis of stationarity around a non-linear trend is preferred to the hypothesis of a unit root. In addition, the null hypothesis of symmetry ( $\rho_1 = \rho_2$ ) is rejected at the conventional significance level for Austria, Luxembourg, Norway, Switzerland and the UK, but not for Sweden. Thus, the evidence suggests that the 'deep' cycles (adjustments) around the threshold value of the consumption-income ratios of Austria, Luxembourg, Norway and the UK are asymmetric.

We turn our attention to the results of the LNV-MTAR model. This model allows the

adjustment to depend on the previous period's change in the consumption-income ratio.

**Table 5: Results of the LNV-TAR unit root test**

	demeaned		demeaned and detrended	
	$F_{\alpha(\beta)}$	$F_A$	$F_{\alpha\beta}$	$F_A$
Austria	11.633*	2.975*	22.388***	8.035***
Belgium	6.707	0.009	10.987	0.287
Canada	5.947	0.032	10.036	0.746
Denmark	3.531	2.764	4.200	0.023
Finland	7.858	1.235	7.651	0.376
France	2.906	4.135**	5.927	0.045
Greece	2.018	1.237	6.793	11.584***
Iceland	10.093	0.082	9.000	0.010
Ireland	6.252	6.496***	8.330	8.847***
Italy	5.253	0.254	10.466	10.47***
Japan	9.941	3.610**	11.276	0.287
Luxembourg	14.643**	23.429***	12.074*	17.092***
Netherlands	7.454	5.961**	7.606	5.211**
New Zealand	10.593	4.523**	10.539	4.521**
Norway	10.796*	14.369***	17.774***	6.155**
Spain	6.931	8.536***	11.109	8.772**
Sweden	11.830*	<0.001	11.766	<0.001
Switzerland	4.733	0.031	14.686**	12.756***
the UK	15.324***	22.023***	17.381***	20.786***
the US	7.545	2.852*	8.290	0.309

(1) \*, \*\*, \*\*\* denote significance at the 10%, 5% and 1%, respectively. (2)  $F_{\alpha(\beta)}$  and  $F_A$  denote the F-statistics for the null hypothesis of a unit root  $H_0 : \rho_1 = \rho_2 = 0$  and symmetry  $H_0 : \rho_1 = \rho_2$ , respectively. (3) The 10%, 5% and 1% critical values for the  $F_{\alpha(\beta)}$  statistic of Model B are 10.62, 12.20 and 15.26, respectively. (4) The 10%, 5% and 1% critical values for the  $F_{\alpha\beta}$  statistic of Model C are 11.86, 13.40 and 16.83, respectively. (5) The numbers in parenthesis are standard errors. (6) The numbers in square parenthesis are p-values.

The results for the test statistics  $F_{\alpha(\beta)}^*$  and  $F_{\alpha\beta}^*$  of the LSTR-MTAR model for Models B and C are reported in the left and right panels of Table 6, respectively. For Austria, Belgium, Japan, New Zealand, Norway, Sweden and the UK, the null hypothesis of a unit root ( $\rho_1 = \rho_2 = 0$ ) in Model C is rejected at the 10 percent or better significance level. The results indicate that the consumption-income ratios for these countries once again exhibit non-linear trend stationarity. However, the null hypothesis of symmetry ( $\rho_1 = \rho_2$ ) is rejected at the 5 percent significance level only for Austria and New Zealand. Thus, it appears that the 'sharpness' cycles (adjustments) around the threshold values of the consumption-income ratios of these countries are asymmetric only for Austria and New Zealand.

All in all, the empirical results for the respective LNV-TAR and LNV-MTAR models reveal that the consumption-income ratios of Austria, Belgium, Japan, Luxembourg, New Zealand, Norway, Sweden, Switzerland and the UK exhibit stationarity after taking into account the non-linear trend, which is in line with the relative income hypothesis, the permanent income hypothesis and the life cycle hypothesis. If consumers are forward-looking and try to smooth

their consumption over time as suggested by the three hypotheses, then the aggregate demand policy should have little effect on the short-run spending (Brady, 2004). Nevertheless, it is model-dependent for the consumption-income ratios to be asymmetrically adjusted after taking the non-linear trend into consideration. It is easy to observe that the estimated logistic smooth transition trend of Model C is quite close to that of the raw data. This plot echoes the rejection of the null hypothesis of a unit root by the  $F_{\alpha(\beta)}$ ,  $F_{\alpha\beta}$ ,  $F_{\alpha(\beta)}^*$  and  $F_{\alpha\beta}^*$  statistics as shown in Tables 5 and 6.

**Table 6: Results of the LNV-MTAR unit root test**

	demeaned		demeaned and detrended	
	$F_{\alpha(\beta)}^*$	$F_A$	$F_{\alpha\beta}^*$	$F_A$
Austria	10.551	0.239	22.879***	3.881**
Belgium	7.233	0.909	12.129*	1.961
Canada	5.933	0.008	9.976	0.253
Denmark	3.335	0.393	4.298	0.223
Finland	9.064	3.284*	9.247	3.698**
France	2.320	0.087	6.008	0.232
Greece	1.862	0.940	7.173	0.243
Iceland	10.311	0.524	9.004	0.015
Ireland	8.444	8.206***	10.164	9.791***
Italy	6.397	2.357	6.982	0.030
Japan	9.762	1.472	12.157*	1.549
Luxembourg	8.251	5.283***	10.155	6.425***
Netherlands	7.089	0.119	7.222	0.099
New Zealand	18.717***	17.682***	18.716***	17.678***
Norway	5.315	1.138	16.406**	0.770
Spain	7.128	2.345	8.556	0.023
Sweden	12.636**	1.278	12.536*	1.161
Switzerland	4.959	0.415	11.381	0.209
the UK	10.440	4.592**	12.374*	0.228
the US	6.410	0.211	8.220	0.421

(1) \*, \*\*, \*\*\* denote significance at the 10%, 5% and 1%, respectively. (2)  $F_{\alpha(\beta)}^*$  and  $F_A$  denote the F-statistics for the null hypothesis of a unit root  $H_0 : \rho_1 = \rho_2 = 0$  and symmetry  $H_0 : \rho_1 = \rho_2$ , respectively. (3) The 10%, 5% and 1% critical values for the  $F_{\alpha(\beta)}^*$  statistic of Model B are 10.75, 12.17 and 15.40, respectively. (4) The 10%, 5% and 1% critical values for the  $F_{\alpha\beta}^*$  statistic of Model C are 12.09, 13.66 and 16.9, respectively. (5) The numbers in parenthesis are standard errors. (6) The numbers in square parenthesis are p-values.

### 3.3 Results of the LNV-ESTAR approaches

In this subsection we test for the null hypothesis of a unit root against the alternative of the structural break non-linearity and size non-linearity at the same time. The size non-linearity which is related to the possibility of an asymmetric speed of adjustment towards equilibrium. That is, the further the consumption-income ratio deviates from its fundamental equilibrium, the faster will be the speed of mean reversion. We adopt the LNV-inf-PS [Eq. (12)] and LNV-KSS

[Eq. (13)] statistics to the consumption-income ratios and report empirical results in Tables 7 and 8, respectively. The critical values of the two tests are obtained via Monte Carlo simulations. The results from the LNV-inf-PS test indicate that we can reject the null of a unit root only in three cases (France, Luxembourg and Sweden) at the 10% significance level or better. When we turn to the INV-KSS test, rejection of the null of a unit root occurs in ten cases (Austria, Ireland, Japan, Luxembourg, Netherlands, New Zealand, Norway, Spain, Switzerland and the UK).

**Table 7: Results of the LNV-inf-PS unit root test**

	LNV-inf-PS		
	Model A	Model B	Model C
Austria	-1.417	-1.187	-0.793
Belgium	-1.978	-2.878	-1.394
Canada	-1.779	-1.642	-2.517
Denmark	-1.122	-1.869	-1.680
Finland	-2.887	-2.777	-2.721
France	-3.022	-4.561**	-2.734
Greece	-2.788	-3.044	-2.125
Iceland	-1.471	-3.365	-1.824
Ireland	-3.256	-3.046	-3.651
Italy	-2.544	-2.743	-2.186
Japan	-2.167	-2.180	-2.343
Luxembourg	-1.885	-4.292*	0.338
Netherlands	-1.713	-1.558	-2.701
New Zealand	0.212	0.140	0.140
Norway	-0.428	-3.675	-3.675
Spain	-3.118	-3.053	-2.040
Sweden	-2.364	-4.123*	-4.106
Switzerland	-2.821	-1.617	-3.515
the UK	-3.069	-1.572	-1.547
the US	-2.851	-3.571	-1.676
10% cv	- 3.320	- 4.005	- 4.344
5% cv	- 3.999	- 4.332	- 4.665
1% cv	- 4.697	- 5.012	- 5.348

(1) \*, \*\*, \*\*\* denote significance at the 10%, 5% and 1% levels, respectively. (2) The critical values for the LNV-inf-PS statistics are obtained via Monte Carlo simulation.

**Table 8: Results of the LNV-KSS unit root test**

	LNV-KSS		
	Model A	Model B	Model C
Austria	-3.361	-3.391	-4.754**
Belgium	-2.565	-0.809	-1.878
Canada	-2.200	-2.863	-2.967
Denmark	-1.499	-2.034	-2.663
Finland	-2.519	-2.402	-2.111
France	-2.160	-0.714	-1.040
Greece	-2.504	-0.796	-2.298
Iceland	-1.768	-2.408	-1.368
Ireland	-3.201	-2.909	-4.044*
Italy	-1.822	-1.821	-3.759
Japan	-3.937**	-4.508**	-4.164*
Luxembourg	-1.150	-4.532**	-4.569**
Netherlands	-3.144	-4.350**	-3.795
New Zealand	-3.355	-4.478**	-4.478**
Norway	-0.771	-4.013*	-4.797**
Spain	-3.558*	-3.501	-3.729
Sweden	-2.160	-3.328	-3.316
Switzerland	-2.046	-2.444	-4.775**
the UK	-1.946	-5.702**	-5.710**
the US	-3.095	-3.276	-4.754**
10% cv	-3.475	-3.766	-4.001
5% cv	-3.834	-4.074	-4.317
1% cv	-4.427	-4.650	-5.030

(1) \*, \*\*, \*\*\* denote significance at the 10%, 5% and 1% levels, respectively. (2) The critical values for the LNV-KSS statistics are obtained via Monte Carlo simulation.

In sum, the results point to the rejection of the null hypothesis of a unit root and favor the alternative of a globally stationary ESTAR process around a non-linear deterministic trend for ten countries. This implies that the size non-linearity is a vital feature of the consumption-income ratios of Austria, Ireland, Japan, Luxembourg, Netherlands, New Zealand, Norway, Spain, Switzerland and the UK. If we overlook this feature, then we will be inclined to reach a spurious conclusion that the consumption-income ratios is a non-stationary process.

Finally, the LNV-Sollis-AESTAR specification of Eq (14) is examined by first testing the null hypothesis of a unit root,  $H_0 : \lambda_1 = \lambda_2 = 0$ , which allows for symmetric or asymmetric non-linear ESTAR adjustment under the alternative hypothesis. The results are included in the left panel of Table 9. It is found that the null hypothesis of a unit root can be rejected at the 10 percent significance level or better in nine countries (Austria, Japan, Luxembourg, Netherlands, New Zealand, Norway, Spain, Switzerland and the UK). The results imply that the consumption-income ratios are symmetric or asymmetric ESTAR non-linear stationary processes for nine of twenty countries. For those countries where it is possible to reject the null hypothesis of a unit root, we then test for the null hypothesis of symmetric ESTAR non-linearity against the alternative of asymmetric ESTAR non-linearity using the auxiliary model (14) by testing  $H_0 : \lambda_2 = 0$  against  $H_1 : \lambda_2 < 0$  with a standard F-test (see the right panel of Table 9). It is found that the null of the symmetric ESTAR non-linearity is rejected at the conventional significance

level for Luxembourg, New Zealand, Norway and Spain, indicating that consumption-income ratios are possibly characterized by the asymmetric ESTAR non-linear stationary process for the four countries. In other words, the speed of mean reversion of consumption-income ratios of these four countries depend not only on the absolute deviation from the equilibrium, but also upon the sign of the shock.

**Table 9: Results of the LNV-Sollis-AESTAR unit root test**

	$H_0 : \lambda_1 = \lambda_2 = 0$			$H_0 : \lambda_2 = 0$		
	Model A	Model B	Model B	Model A	Model B	Model B
Austria	6.228	6.117	11.686**	1.136	0.884	0.834
Belgium	3.234	1.377	1.852	0.004	2.088	0.226
Canada	2.772	4.034	5.219	0.728	0.013	1.549
Denmark	1.105	2.371	3.489	0.003	0.633	0.010
Finland	3.892	2.957	2.589	1.180	0.225	0.739
France	2.856	2.906	1.263	1.043	5.264**	1.436
Greece	3.231	1.915	5.572	0.274	3.173*	0.384
Iceland	1.535	4.336	1.173	0.001	2.699	0.491
Ireland	5.113	4.363	8.163	0.134	0.359	0.241
Italy	1.691	2.904	9.005	1.608	2.408	3.305*
Japan	8.838**	10.526**	12.097**	1.926	0.801	0.807
Luxembourg	0.759	15.220**	9.997*	0.213	7.540**	0.013
Netherlands	5.479	9.511**	7.037	1.060	0.321	0.0002
New Zealand	9.841**	10.206**	10.207**	7.198**	0.525	0.526
Norway	2.079	9.245**	17.928**	3.530*	2.076	9.441**
Spain	7.195*	6.886	9.993*	1.598	1.424	5.078*
Sweden	2.430	5.620	5.576	0.253	0.396	0.293
Switzerland	2.553	5.229	11.224**	0.786	0.531	0.029
the UK	1.861	16.563**	16.476**	0.001	0.749	0.574
the US	4.995	5.293	3.417	0.491	0.033	1.328
10% cv	6.966	7.923	9.047			
5% cv	8.310	9.238	10.365			
1% cv	10.682	11.760	13.249			

(1) \*, \*\*, \*\*\* denote significance at the 10%, 5% and 1% levels, respectively. (2) The critical values for the LNV-Sollis-SESTAR statistics are obtained via Monte Carlo simulation.

### 3.4 Comparison of results with selective literature

Some comparisons with previous studies have been drawn from our empirical tests. First of all, the empirical results of this paper do not agree with Sarantis and Stewart (1999), Tsionas and Christopoulos (2002), Cook (2002, 2003, 2005), Romero-Avila (2008), Fallahi (2012) and Cerrato et al. (2013). These papers provide empirical evidence that the consumption-income ratios of the OECD countries are non-stationary by employing the unit root test with structural breaks or the panel unit root tests, which are in favor of the Keynesian absolute income hypothesis and Deaton's (1977) involuntary savings theory.<sup>1</sup> A weakness of these studies is that they overlook non-linearity or asymmetry in testing for the null hypothesis of a unit root.

<sup>1</sup> Readers are referred to Table 1 for details.

Therefore, they are inclined to accept the result of non-stationarity if the non-linearity really exists in the data.

Second, the empirical results of this paper are in conformity with Romero-Avila (2009), Liao (2010) and Elmi and Ranjbar (2013). In particular, the results obtained from Romero-Avila (2009) show that the application of the panel stationarity test with multiple breaks supports the existence of regime-wise stationarity in OECD consumption-income ratios. Empirical results from Elmi and Ranjbar (2013) based on using the flexible non-linear stationarity test show that the mean reversion hypothesis is not rejected for 12 of 16 OECD countries. Our results have important theoretical implications because they are in complete accord with the validity of Duesenberry's (1952) relative income hypothesis, Friedman's (1957) permanent income hypothesis and Ando and Modigliani's (1963) life-cycle hypothesis, which all predict the existence of an equilibrium relationship between consumption and income, thus implying stationarity in the consumption-income ratio. Moreover, our results lend support to a key prediction of the "great-ratios" literature that advocates the existence in the long run of a stationary consumption-income ratio along the balanced growth path.

#### **4. Concluding remarks**

Whether or not the average propensity to consume (APC) is a non-stationary process has important theoretical implications. If the APC exhibits mean reversion, then it is implied that there exists an equilibrium relationship between consumption and income. This paper attempts to revisit the non-stationarity of the consumption-income ratios of twenty OECD countries. A variety of unit root tests ranging from univariate estimators to non-linear testing principles are employed in an effort to obtain inferences that are robust to problems associated with non-stationarity. We adopt the TAR, MTAR, LNV-ADF, LNV-TAR, LNV-MTAR and various LNV-ESTAR unit root tests which help detect the non-linear consumption-income relationship without specifying the threshold in advance.

For the benefit of readers, we summarize our empirical results in Tables 10. This study reaches the following key conclusions. First, by using a battery of univariate unit root tests, the consumption-income ratios of these OECD countries are characterized by a unit root process, which is not in violation of the Keynesian absolute income hypothesis and Deaton's (1977) involuntary savings theory. Second, empirical evidence from the LNV-TAR and LNV-MTAR tests shows that the consumption-income ratios of Austria, Belgium, Japan, Luxembourg, New Zealand, Norway, Sweden, Switzerland and the UK are stationary after taking account of the non-linear trend. Third, results from the LNV-ESTAR approaches show that asymmetric speed of adjustment (size non-linearity) is a vital feature of the consumption-income ratios of Austria, Ireland, Japan, Luxembourg, Netherlands, New Zealand, Norway, Spain, Switzerland and the UK. In sum, our empirical results are congruent with the theoretical predictions of the relative income hypothesis, the permanent income hypothesis and the life-cycle hypothesis.

**Table 10: Summary of a variety of the LNV-ESTAR unit root tests**

	SB non-linearity	SB and sign non-linearities	SB and size non-linearities	SB, size and sign non-linearities		
Countries	LNV-ADF	LNV-TAR	LNV-MTAR	LNV-inf-pS	LNV-KSS	LNV-Sollis-AESTAR
Austria	yes	yes (yes)	yes (yes)		yes	yes (no)
Belgium	yes		yes			
Canada	yes					
Denmark						
Finland						
France				yes		
Greece						
Iceland	yes					
Ireland					yes	
Italy						
Japan			yes		yes	yes (no)
Luxembourg		yes (yes)		yes	yes	yes (yes)
Netherlands					yes	yes (no)
New Zealand	yes		yes (yes)		yes	yes (yes)
Norway	yes	yes (yes)	yes		yes	yes (yes)
Spain					yes	yes (yes)
Sweden	yes	yes	yes	yes		
Switzerland	yes	yes (yes)			yes	yes (no)
the UK	yes	yes (yes)	yes		yes	yes (no)
the US	yes					

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