

under the forward measure $Q^{T_{i+1}}$ as follows:

$$IDIC(\tau, T_{i+1}; T_{ij}; K_{ij}) = B_n(\tau, T_{i+1}) E^{Q^{T_{i+1}}} \left(\frac{X(T_i)}{X(0)} R(T_i; T_i) I(R(T_{ij}; T_{ij})) > K_{ij} \mid \mathcal{F}_\tau \right). \tag{B.1}$$

According to (1), we can rewrite (B.1) as follows:

$$\begin{aligned} \text{(B.1)} &= \frac{B_n(\tau, T_{i+1})}{X(0)} E^{Q^{T_{i+1}}} \left(X(T_i) \left(\frac{1 - B_n(T_i, T_i^*)}{\Gamma_i B_n(T_i, T_i^*)} \right) I(B_n(T_{ij}, T_{ij}^*) < \bar{K}_{ij}) \mid \mathcal{F}_\tau \right), \\ &= \frac{B_n(\tau, T_{i+1})}{\Gamma_i X(0)} E^{Q^{T_{i+1}}} \left(\frac{X(T_i) B_r(T_i, T_i)}{B_n(T_i, T_i^*)} I(B_n(T_{ij}, T_{ij}^*) < \bar{K}_{ij}) \mid \mathcal{F}_\tau \right) - \\ &\quad \frac{B_n(\tau, T_{i+1})}{\Gamma_i X(0)} E^{Q^{T_{i+1}}} \left(X(T_i) I(B_n(T_{ij}, T_{ij}^*) < \bar{K}_{ij}) \mid \mathcal{F}_\tau \right). \end{aligned} \tag{B.2}$$

The second part in (B.2) can be obtained by Appendix A as follows:

$$\frac{B_n(\tau, T_{i+1})}{\Gamma_i X(0)} E^{Q^{T_{i+1}}} \left(X(T_i) I(B_n(T_{ij}, T_{ij}^*) < \bar{K}_{ij}) \mid \mathcal{F}_\tau \right) = \frac{IDC(\tau, T_{i+1}; T_{ij}; K_{ij})}{\Gamma_i}. \tag{B.3}$$

Next, we rewrite the first part in (B.2) as follows:

$$\frac{B_n(\tau, T_{i+1})}{\Gamma_i X(0)} E^{Q^{T_{i+1}}} \left(g^*(T_i, T_i) I(h(T_{ij}, T_{ij}) < \bar{K}_{ij}) \mid \mathcal{F}_\tau \right) \tag{B.4}$$

where $h(T_{ij}, T_{ij})$ is defined in (A.14) and $g^*(t, T_i)$ is defined as follows:

$$g^*(t, T_i) = \frac{X(t) B_r(t, T_i)}{B_n(t, T_{i+1})} \frac{B_n(t, T_{i+1})}{B_n(t, T_i^*)}.$$

Similar to the deriving process of IDC in Appendix A, (B.4) can be derived by replacing $g(t, T_i)$ with $g^*(t, T_i)$ and the result is given as follows:

$$\text{(B.4)} = \frac{X(\tau) B_r(\tau, T_i)}{X(0) B_n(\tau, T_i)} \frac{B_n(\tau, T_i)}{\Gamma_i B_n(\tau, T_i^*)} \varnothing(\tau, T_i) B_n(\tau, T_i) N(d^*(T_{ij})). \tag{B.5}$$

where

$$d^*(T_{ij}) = \frac{\ln\left(\frac{1 + \Gamma_{ij} R(\tau, T_{ij})}{1 + \Gamma_{ij} K_{ij}}\right) - \xi^*(\tau, T_{ij}) + \frac{1}{2} V(\tau, T_{ij})}{\sqrt{V(\tau, T_{ij})}}, \tag{B.6}$$

$$\xi^*(\tau, T_{ij}) = \int_\tau^{T_{ij}} (\mu_h(\mathbf{u}, T_{ij}) + \eta^*(\mathbf{u}, T_{ij}) \cdot \mathbf{v}_h(\mathbf{u}, T_{ij})) d\mathbf{u}, \tag{B.7}$$

$$\eta^*(\mathbf{u}, T_{ij}) = \begin{cases} \mathbf{v}_{g^*}(\mathbf{t}, T_i) & 0 \leq t \leq T_i, \\ \mathbf{0} & T_i < t \leq T_{ij}, \end{cases} \tag{B.8}$$

$$\mathbf{v}_{g^*}(\mathbf{t}, T_i) = (\sigma_x(\mathbf{t}) + \sigma_{B_n}(\mathbf{t}, T_i^*) - \sigma_{B_r}(\mathbf{t}, T_i)), \tag{B.9}$$

$$\varnothing^*(\tau, T_i) = \exp\left(\int_\tau^{T_i} \sigma_{B_n}(\mathbf{u}, T_{i+1}) - \sigma_{B_n}(\mathbf{u}, T_i^*) \cdot (\sigma_{B_r}(\mathbf{u}, T_i) - \sigma_{B_n}(\mathbf{u}, T_i^*) - \sigma_x(\mathbf{u}))\right). \tag{B.10}$$

With the above derivation, IDIC can be derived as follows:

$$IDIC(\tau, T_{i+1}; T_{ij}; K_{ij}) = \text{(B.5)} - \text{(B.3)}.$$

As to the pricing formula for the IDIP, it can be derived similarly to the IDIC case.